squared error: $J_i(n) = \left(\overline{P}_R(d_i, n) - P_{R,a}(d_i) \right)^2$

$$\overline{P}_R(d_0, n) = \overline{P}_R(d_0) = 10\log_{10}\left(P_T G_T G_R \left(\frac{4\pi\lambda}{d_0}\right)^2 \right)$$

Assume that the model has minimal variation around the predicted mean value at d_0 $\rightarrow$ $\overline{P}_R(d_0) \simeq P_{R,a}(d_0)$

Note that $d_1 = 100\text{m} = d_0 \rightarrow \overline{P}_R(d_0)$: known

$\overline{P}_R(d_0) = 0$ dBm

$$\therefore \quad \overline{P}_R(d_i, n) = 0 + 10n\log\left(\frac{d_0}{d_i}\right)^n$$

$$J_i(n) = \left(10n\log\left(\frac{d_0}{d_i}\right)^n - P_{R,a}(d_i) \right)^2 = \begin{cases} (0-0)^2, & i=1 \\ (-3n-(-20))^2, & i=2 \\ (-10n-(-35))^2, & i=3 \\ (-14.77-(-70))^2, & i=4 \end{cases}$$

$$\text{MSE} = \frac{1}{4}\sum_{i=1}^{4} J_i(n) = \frac{1}{4}J(n)$$

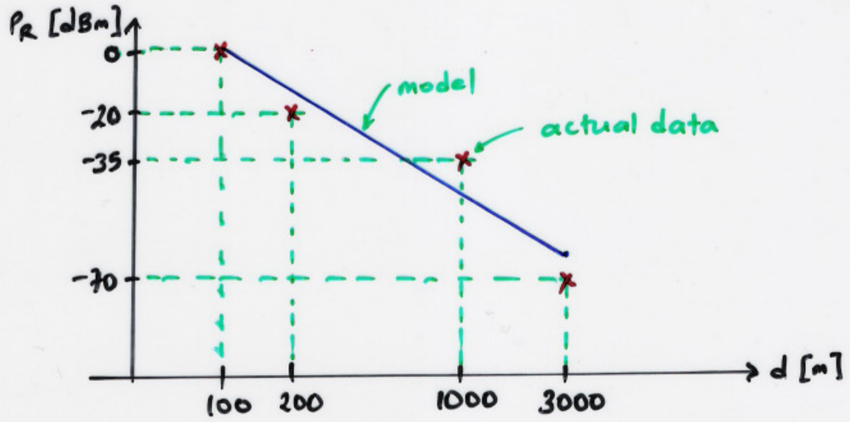
MMSE estimate of n: set $\frac{\partial}{\partial n}\frac{1}{4}J(n) = \frac{\partial}{\partial n}J(n)$ equal to zero

$\rightarrow n = 4.4$

(b) $\frac{1}{4}J(n=4.4) = 38.09$: variance of variation which is modelled by Gaussian distribution : $\sigma^2 = 38.09$ [dB] $\rightarrow$ $\sigma = 6.17$ [dB]

$$P_R(d) = 10\log_{10}\left(P_T G_T G_R \left(\frac{4\pi\lambda}{d_0}\right)^2 \right) + 10n\log_{10}\left(\frac{d_0}{d}\right) + x_\sigma$$

$\hookrightarrow G(0; \sigma = 6.17)$



(c) $\overline{P}_R(d=2000\text{m}) = \underbrace{\overline{P}_R(d_0)}_{0\text{ dBm}} + 10\underbrace{n}_{4.4}\log_{10}\left(\frac{\overset{100}{d_0}}{2000}\right)$

$= 44\log_{10}(1/20) = -57.24\text{ dBm}$

(d) $\text{Prob}\left[\underbrace{P_R(d=2000)}_{\overline{P}_R(d)+x_\sigma} > -60\text{dBm}\right] = Q\left(\frac{-60-(-57.24)}{6.17}\right)$

$\overline{P}_R(d) \to -57.24$, $x_\sigma \to G(0;\sigma=6.17)$

$\underbrace{\overline{P}_R(d)+x_\sigma}_{G(-57.24;\ \sigma=6.17)}$

$= Q(-0.44)$

$= 1 - Q(0.447)$

$= 0.674 = 67.4\%$

(e) $\sigma/n = 6.17/4.4 = 1.40$, $\text{Prob}(P_R(d=2000\text{m}) > -60\text{dBm}) = 67.4\%$

From Fig. 4.18 (Rappaport, 2nd ed.)

→ Percentage of area within a 2000m radius that has $P_R > -60\text{dBm}$

$= 92\%$