* **Shadowing:**

Large-scale variations: to account for the discrepancies between the measured data and the average large-scale path-loss

→ a statistical model

$$P_R = EIRP \times G_R \times \frac{1}{L_p} \times y$$

y → random variable

L_p → deterministic

$$P_R(d)\,[dBm] = \bar{P}_R(d)\,[dBm] + x_\sigma[dB]$$

pay attention to the units!

$\bar{P}_R(d)$ → $P(d_0) + 10n\log_{10}\left(\frac{d_0}{d}\right)$

x_σ → modeled as a Gaussian r.v. with 0 dB mean and σ dB s.d.

$x: G(0;\sigma)$

* parameters d_0, n, σ capture the effects of the propagation environment.

$PL(d_0)$: free-space path-loss at the reference distance d_0

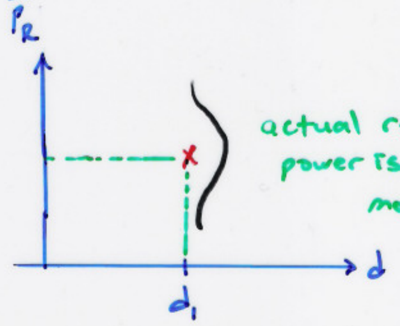
often used values:

microwave links	$d_0 = 1\,km$
outdoor mobile	$d_0 = 100\,m$
microcell	$d_0 = 10\,m$
indoors	$d_0 = 1\,m$

* one can always come up with a more elaborate model; but simplicity is preferred as long as the model is

a) correct in the first-order, and

b) flexible enough to accommodate a variety of environments



actual received power is a random quantity modeled by gaussian distribution

Lognormal RV

given x and $f_X(x)$, find $f_Y(y)$ if $y=g(x)$.

* Fundamental theorem from Probability Theory:
if $x_1, \dots, x_k$ are the solutions of $g(x)$, then

$$f_Y(y) = \sum_{i=1}^{k} \frac{f_X(x_i)}{|g'(x_i)|}$$

Ex 1: $y = g(x) = ax+b$

$x_1 = \frac{y-b}{a}$; $g'(x) = a$, $g'(x_1) = a$ $\rightarrow$ $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$

Ex 2: $y = ax^2$ (assume $a>0$)

$x_1 = \sqrt{y/a}$
$x_2 = -\sqrt{y/a}$ $\quad g'(x) = 2ax$

$$f_Y(y) = \frac{f_X(\sqrt{y/a})}{|2a\sqrt{y/a}|} + \frac{f_X(-\sqrt{y/a})}{|-2a\sqrt{y/a}|} = \frac{1}{2\sqrt{ay}}\left(f_X(\sqrt{y/a}) + f_X(-\sqrt{y/a})\right)$$

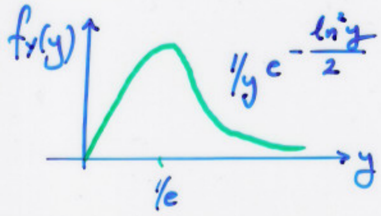
Ex 3: $y = e^x = g(x)$ $\qquad x: G(\eta;\sigma)$

$x_1 = \ln y$

$g'(x) = e^x = y$ $\rightarrow$ $f_Y(y) = \frac{1}{y} f_X(\ln y)$

Note: $y>0$

$$= \frac{1}{y\sigma\sqrt{2\pi}} e^{-(\ln y - \eta)^2/2\sigma^2}$$



Lognormal r.v.: $x: G(0; \sigma)$

$$x = 10\log_{10} y \rightarrow y = 10^{x/10} = g(x)$$

$y \rightarrow$ lognormal

$\rightarrow x_1 = 10\log_{10} y$

$$\ln y = \frac{x}{10}\ln 10$$

$$\rightarrow y = e^{\ln y} = e^{\frac{\ln 10}{10}x}$$

$$g'(x) = \underbrace{e^{\frac{\ln 10}{10}x}}_{y} \cdot \frac{\ln 10}{10} = y\,\frac{\ln 10}{10}$$

$$\therefore f_Y(y) = \frac{f_x(10\log_{10} y)}{y\,\frac{\ln 10}{10}} = \frac{1}{\left(\sqrt{2\pi}\,\frac{\ln 10}{10}\right)\sigma y}\, e^{-\frac{(10\log_{10} y)^2}{2\sigma^2}}$$

Note: $\sigma = \sqrt{var(x)}$

$\sigma \neq \sqrt{var(y)}$

$E[y]$ → lognormal r.v.

σ_x	$E[y]$	σ_y
0.01	1.00003	0.0023
0.1	1.0003	0.023
1	1.03	0.24
2	1.11	0.541
5	1.94	3.22
10	14.2	148.9
20	22396	3.2×10^6

Note that

if $y = g(x) \rightarrow E[y] \neq g(E[x])$

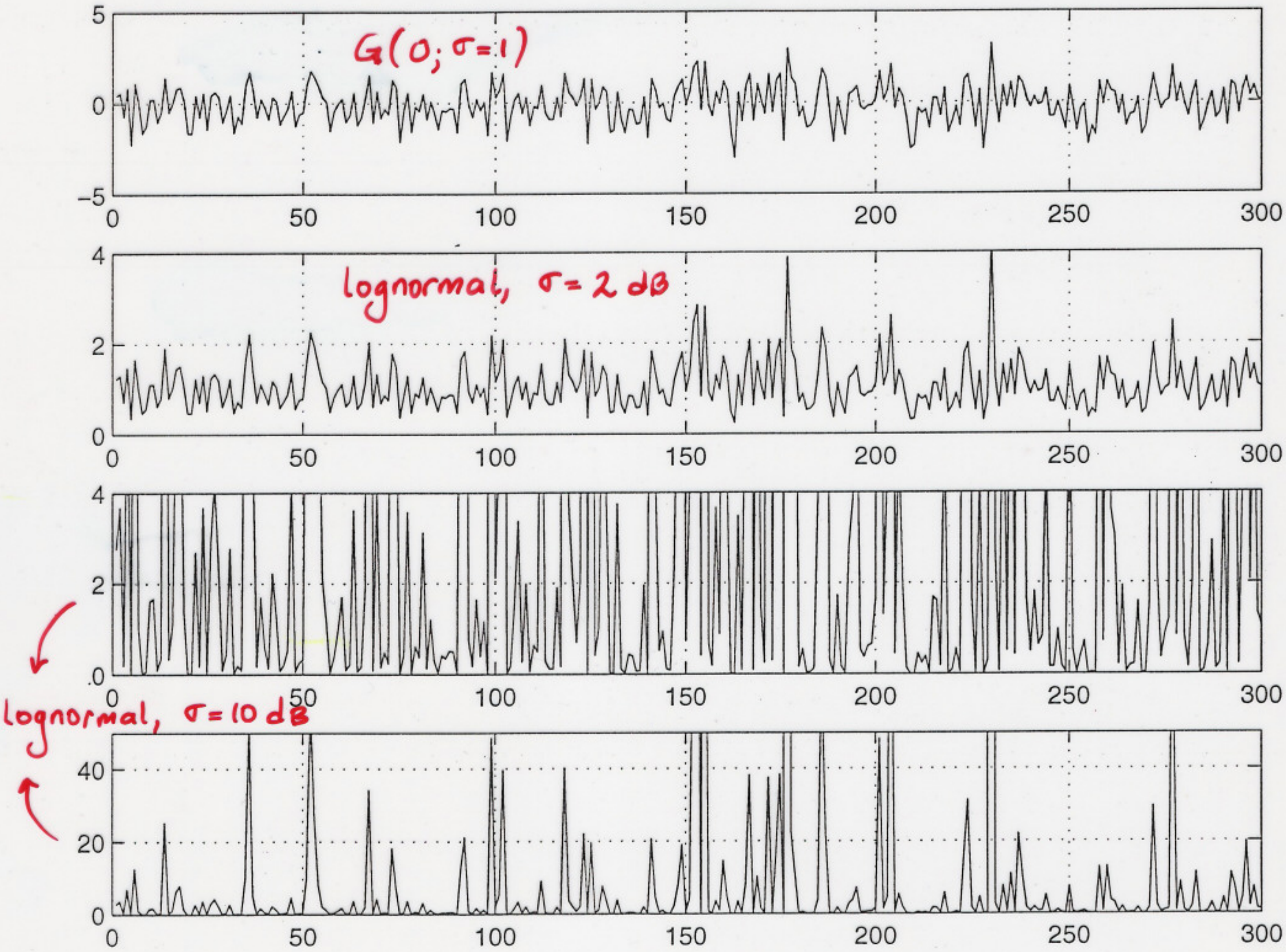
$$E[y] = \int_{-\infty}^{\infty} g(x) f_x(x)\,dx$$

In our case,

$$E[y] = \int_{-\infty}^{\infty} 10^{x/10} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}}\,dx$$

: a function of σ_x

$x = 10\log_{10} y \equiv y = 10^{x/10}$



$s = \sigma$

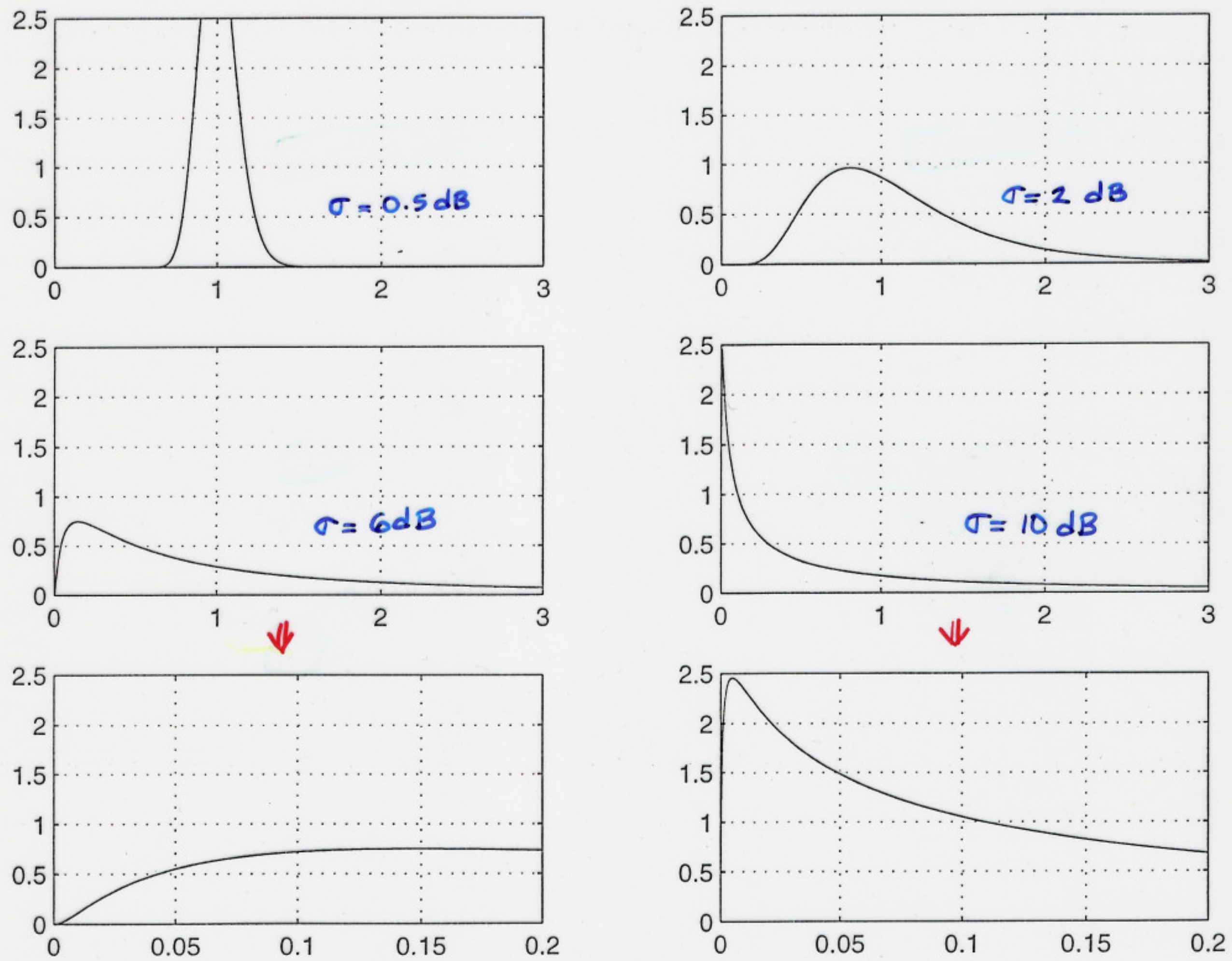
```
%% matlab code to generate lognormal rv

n=1:1:300;
x = randn(1,300); %% x: Gaussion with u=0, std=s=1
subplot(4,1,1), plot(n,x), grid on

y = 10.^(2*x/10); %% y: lognormal (s=2)
subplot(4,1,2), plot(n,y), grid on, axis([0 300 0 4])

y = 10.^(10*x/10); %% y: lognormal (s=10)
subplot(4,1,3), plot(n,y), grid on, axis([0 300 0 4])
subplot(4,1,4), plot(n,y), grid on, axis([0 300 0 50]) %% detail
```

Lognormal PDFs

$y = 10^{x/10}$, $x \sim G(0;\sigma)$

$\sigma \neq std(y)$

$\sigma = std(x)$




```
%% matlab code to generate lognormal pdf

%% y: rv denoting the effect of shadow fading (linear scale)
%% x = 10*log10(y) ... Gaussian with u=0, std=s

s = 0.5;
y = 0.0001:0.0001:3;
x = exp(-((10*log10(y)).^2)/(2*s^2))./(y*s*sqrt(2*pi)*log(10)/10);
subplot(3,2,1), plot(y,x), axis([0 3 0 2.5]), grid on

s = 2;
y = 0.0001:0.0001:3;
x = exp(-((10*log10(y)).^2)/(2*s^2))./(y*s*sqrt(2*pi)*log(10)/10);
subplot(3,2,2), plot(y,x), axis([0 3 0 2.5]), grid on

s = 6;
y = 0.0001:0.0001:3;
x = exp(-((10*log10(y)).^2)/(2*s^2))./(y*s*sqrt(2*pi)*log(10)/10);
subplot(3,2,3), plot(y,x), axis([0 3 0 2.5]), grid on
subplot(3,2,5), plot(y,x), axis([0 .2 0 2.5]), grid on

s = 10;
x = exp(-((10*log10(y)).^2)/(2*s^2))./(y*s*sqrt(2*pi)*log(10)/10);
subplot(3,2,4), plot(y,x), axis([0 3 0 2.5]), grid on
subplot(3,2,6), plot(y,x), axis([0 .2 0 2.5]), grid on
```

$\sigma = s$