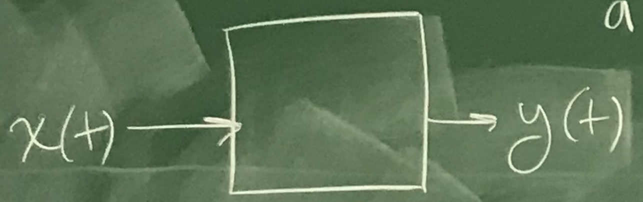
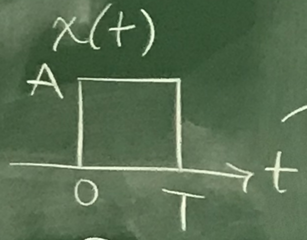
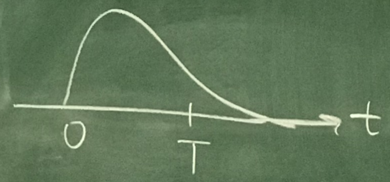
Input-Output Characterization of a LTI System

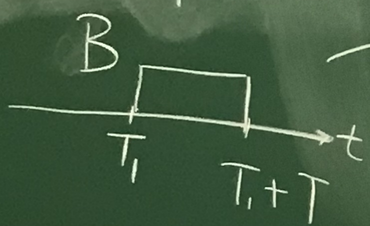


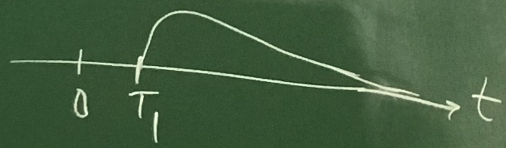
ex: channel

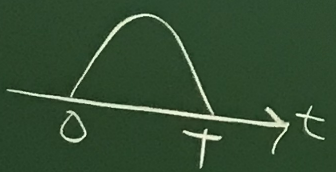
09 Jan 2020









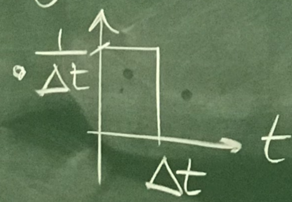


Impulse (Delta) Function

many ways of defn.

- $\delta(t) = \begin{cases} 0, & t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)\,dt = 1 \end{cases}$

- $g(t)$



$$\delta(t) = \lim_{\Delta t \to 0} g(t)$$

- convolution operation

$$x(t) * \delta(t) \triangleq \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

arbitrary function

$$= \int_{-\infty}^{\infty} x(t-\tau)\delta(\tau)\,d\tau = x(t)$$

- impuls
func
ope

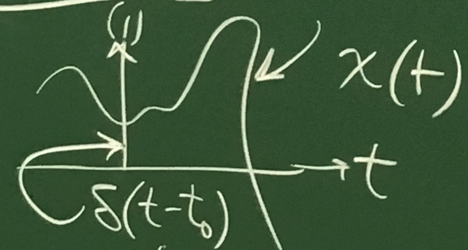
- Uni

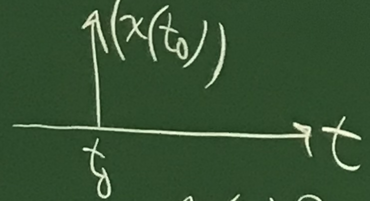
$\int_{-\infty}^{\infty} x($

- impulse function is 'identity' function wrt convolution operation.

- Unique $(*)$

$$\int_{-\infty}^{\infty} x(t-\tau)\,\delta(\tau)\,d\tau \to \int x(t)\,\delta(\tau)\,d\tau = x(t)\int \delta(\tau)\,d\tau = x(t)$$



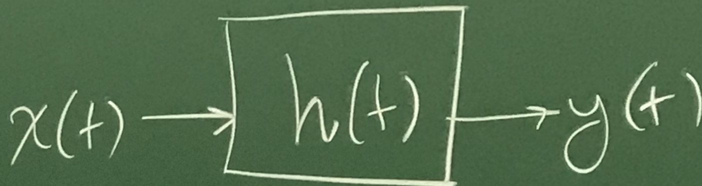


$x(t_0)\,\delta(t-t_0)$

$d\tau$

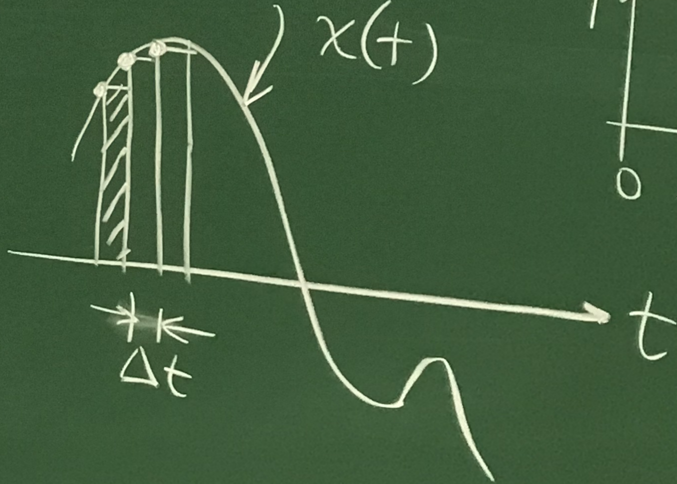
$)\,d\tau = x(t)$

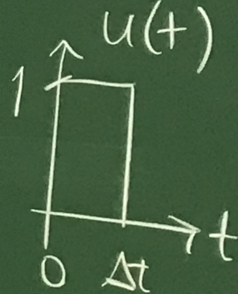
LTI



$x(t) = \delta(t)$

$\to\ y(t) = h(t)$



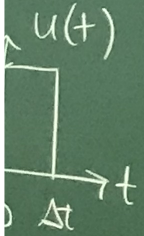


$x(t$

$$x(t) = \lim_{\Delta t \to 0} \sum_{n=-\infty}^{\infty} x(n\Delta t)\, u(t - n\Delta t)$$

$$= \lim_{\Delta t \to 0} \sum_{n=-\infty}^{\infty} x(n\Delta t)\, \delta(t - n\Delta t)\, \Delta t$$

$$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$



t

$$\downarrow$$

$$\boxed{h(t)} \to y(t)$$

$$y(t) = \lim_{\Delta t \to 0} \sum_{n=-\infty}^{\infty} x(n\Delta t)\, h(t - n\Delta t)\, \Delta t$$

$$= \int x(\tau)\, h(t-\tau)\, d\tau$$

det

$$x(t) \to \boxed{h(t)} \to y(t) = x(t) * h(t)$$

time domain

$$X(f) \to \boxed{H(f)} \to Y(f) = X(f)H(f)$$

random $S_x(f) \to \boxed{H(f)} \to S_Y(f)$

$$= S_x(f)\, |H(f)|^2$$