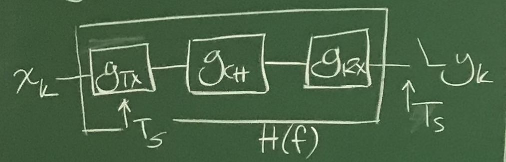
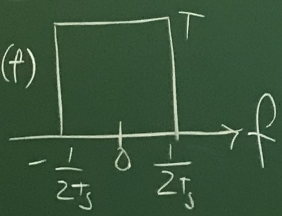
04 Feb 2020

Nyquist no-ISI criterion



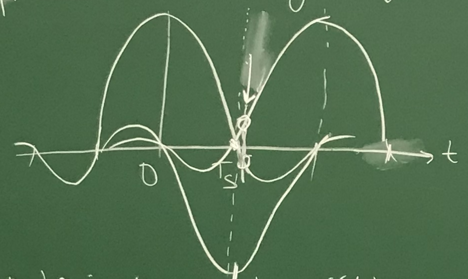
$$\sum_k H\left(f+\frac{k}{T_s}\right) = T_s$$

Min BW soln



$$h(t) = \text{sinc}(t/T_s)$$

A practical concern (engineering)

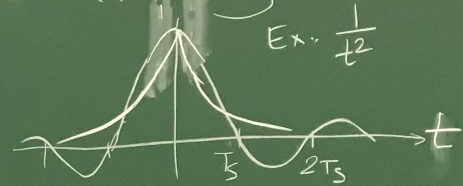


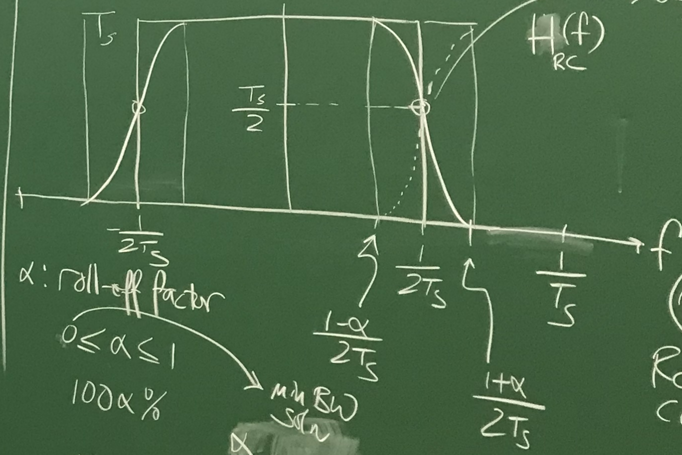
synchronization problem, jitter, clock mismatch, drift, ...

- sinc() does not die out fast enough
- Around the first null: problematic
- New $h(t)$'s which are no-ISI, but die out faster.

$\text{sinc}(t/T_s) \cdot f(t)$ ... guarantee no-ISI

Ex.. $\frac{1}{t^2}$





odd symmetric

α: roll-off factor

$0 \le \alpha \le 1$

$100\alpha\%$

min BW soln

$$\text{Excess BW} = \frac{\alpha}{2T_s}$$

$$\text{BW} = \frac{1+\alpha}{2T_s}$$

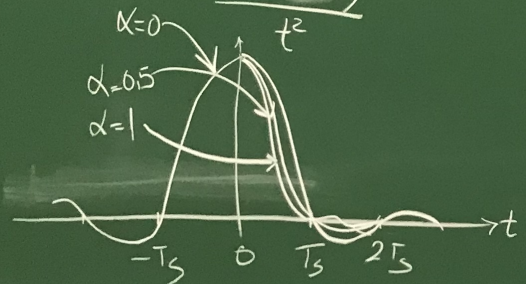
RC

Raised Cosine

$$H_{RC}(f) = \begin{cases} T_s, & |f| \le \frac{1-\alpha}{2T_s} \\ & \frac{1-\alpha}{2T_s} < |f| \le \frac{1+\alpha}{2T_s} \\ 0, & |f| > \frac{1+\alpha}{2T_s} \end{cases}$$

$$\frac{T_s}{2}\left[1+\cos\left(\frac{\pi T_s}{\alpha}\left[|f| - \frac{1-\alpha}{2T_s}\right]\right)\right]$$

$$h_{RC}(t) \sim \frac{\mathrm{sinc}(t/T_s)}{t^2}$$



SISO

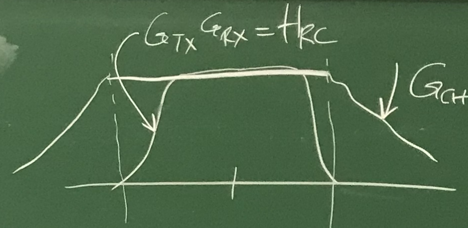
$$SE_s \triangleq \frac{R_s}{B} = \begin{cases} \frac{R_s}{\frac{R_s}{2}(1+\alpha)} = \frac{2}{1+\alpha}, & \text{baseband} \\ \frac{1}{1+\alpha}, & \text{bandpass} \end{cases}$$

- $H_{RC}(f)$ ✓

$$G_{TX}(f)\,G_{CH}(f)\,G_{RX}(f) = H_{RC}(f)$$

Assume $G_{CH}(f) = 1$

Assume $G_{CH}(f) = \begin{cases} 1, & G_{TX}G_{RX} \neq 0 \\ \text{whatever}, & G_{TX}G_{RX} = 0 \end{cases}$



- $G_{TX}(f)\,G_{RX}(f) = H_{RC}(f)$
- Background noise

$g_{TX}(t), g_{RX}(t)$: Matched filter pair
optimize noise performance

$$G_{RX}(f) = G^{*}_{TX}(f) \rightarrow |G_{RX}(f)| = |G_{TX}(f)|$$

$$|G_{RX}|\,|G_{TX}| = |H_{RC}|$$

$$|H_{RRC}(f)| = |G_{RX}(f)| = |G_{TX}(f)| = \sqrt{|H_{RC}(f)|}$$

root raised cosine
square root raised cos (SRRC)

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Nyquist no-ISI criterion

$$x_k \to \boxed{\boxed{g_{Tx}} \to \boxed{g_{CH}} \to \boxed{g_{Rx}}}_{H(f)} \to y_k$$

$\uparrow T_s$ $\uparrow T_s$

$$\sum_k H\left(f + \frac{k}{T_s}\right) = T_s$$

$$x_k \to \boxed{\sqrt{|H_{RC}(f)|}} \to \boxed{|G_{CH}(f)|} \to \oplus \xleftarrow{} w(t) \to \boxed{\sqrt{|H_{RC}(f)|}} \to y_k \to \boxed{\ } \to \hat{a}_k$$

For an arbitrary $G_{CH}(f)$

no-ISI cond is not valid anymore

$$\sqrt{|H_{RC}|}\,|G_{CH}|\,\sqrt{|H_{RC}|} = |H_{RC}|\,|G_{CH}| \quad ✓$$

need an equalizer to "undo" the channel

$$\cdots \to \boxed{\sqrt{|H_{RC}(f)|}} \to \boxed{H_{eq}(f)} \to$$

$$\sqrt{|H_{RC}|}\cdot|G_{CH}|\cdot\sqrt{|H_{RC}|}\cdot|H_{eq}| : ✓$$

$$\text{if } |H_{eq}| = \frac{1}{|G_{CH}|}$$

then,

$$\boxed{\boxed{\ } - \boxed{\ } - \boxed{\ } - \boxed{H_{eq}}}_{H_{RC}} \ ✓$$

Zero-forcing equalization

(complete removal of ISI)

penalty: noise amplification

$$\to \boxed{\sqrt{|H_{RC}|}} \to \boxed{|H_{CH}|} \to \oplus \to \boxed{\sqrt{|H_{RC}|}} \to \boxed{\frac{1}{|H_{CH}|}} \to$$

matched?

$$\sqrt{|H_{RC}|}\,|H_{CH}| \quad \longleftrightarrow \quad \sqrt{|H_{RC}|}\cdot\frac{1}{|H_{CH}|} \quad \times \text{ not matched}$$

Compromise.

Design $H_{eq}(f)$

such that it achieves a balance between supressing ISI and not amplifying noise too much.

MMSE eql.

min mean-square error