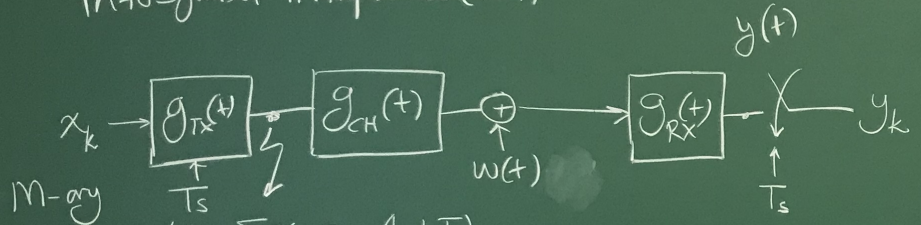
23 Jan 2020

Intersymbol Interference (ISI)



$$x(t) = \sum_k x_k g_{Tx}(t - kT)$$

Perfect transmission $\rightarrow$ $y_k = x_k$, $\forall k$

Assume $w(t) = 0$

$$\rightarrow h(t) = g_{Tx}(t) * g_{CH}(t) * g_{Rx}(t)$$

Cont: $y(t) = \sum_k x_k h(t - kT_s)$

disc:

$$y_l = y(lT_s) = \sum h((l-k)T_s)\, x_k$$

$$y[l] = \sum h[l-k]\, x[k]$$

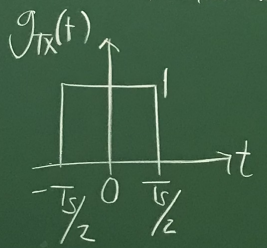
Ex. Ideal channel

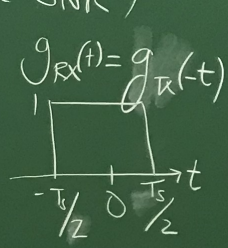
$$g_{CH}(t) = \beta\, \delta(t - t_p)$$

w.l.o.g. $\beta = 1$, $t_p = 0$

$$\rightarrow g_{CH}(t) = \delta(t)$$

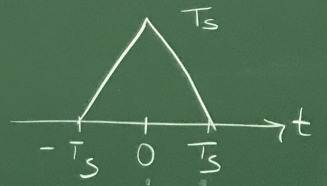
(If there were AWGN, g_{Rx} should be matched to g_{Tx} $\rightarrow$ maximize SNR)



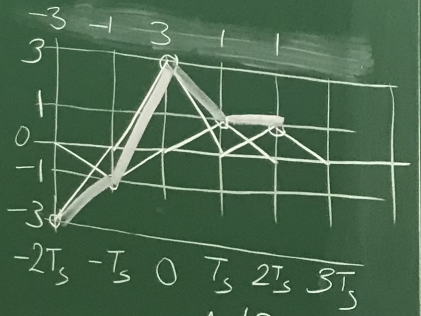


$$h(t) = g_{Tx}(t) * \delta(t) * g_{Rx}(t)$$

$$= g_{Tx}(t) * g_{Rx}(t)$$



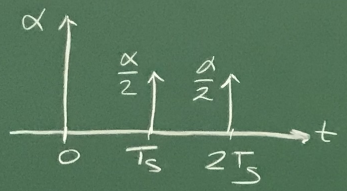
4-ary: $\{-3, -1, 1, 3\}$



no ISI

Ex: 3-path channel

$$g_{CH}(t) = \alpha\delta(t) + \frac{\alpha}{2}\delta(t-T_s) + \frac{\alpha}{2}\delta(t-2T_s)$$

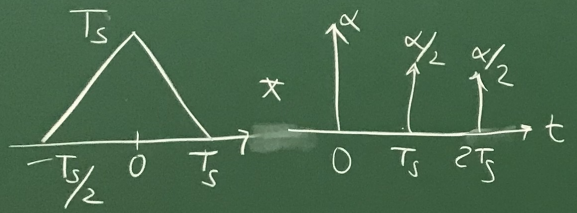


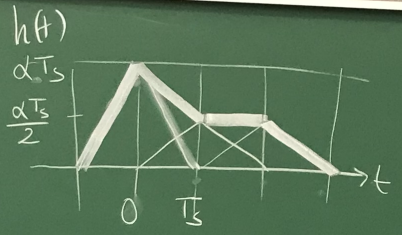
$$\alpha^2 + \frac{\alpha^2}{4} + \frac{\alpha^2}{4} = 1$$

$$\alpha = \sqrt{\frac{2}{3}}$$

$$h(t) = g_{Tx} * g_{CH} * g_{Rx}$$

$$= g_{Tx} * g_{Rx} * g_{CH}$$





$$y_\ell = \sum h_{\ell-k} x_k \quad \text{LTI}$$

$$h(t)\sum_k \delta(t-kT_s) = \delta(t) \quad \text{no-ISI condition}$$

$$H(f) * \frac{1}{T_s}\sum \delta\left(f-\frac{k}{T_s}\right) = 1$$

$$\sum_k H\left(f-\frac{k}{T_s}\right) = T_s \quad \text{no-ISI condition}$$

$\ell=1$: $y_1 = \cdots + h_2 x_{-1} + h_1 x_0 + h_0 x_1 + h_{-1} x_2 + h_{-2} x_3 + \cdots$

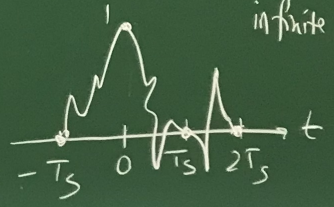
Want $= x_1$

$\rightarrow h_0 = 1$

$h_k = 0,\ k \neq 0$

$$h(t) = \begin{cases} 1, & t=0 \\ 0, & t = kT_s,\ k = \pm1, \pm2, \ldots \end{cases} \rightarrow \text{Nyquist's no ISI condition}$$

infinite possibilities



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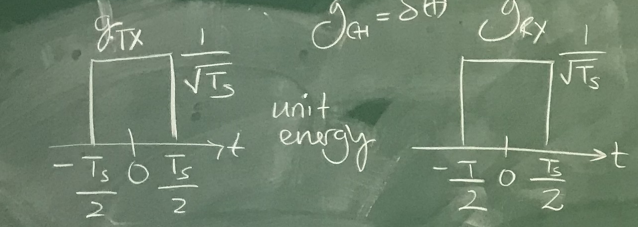
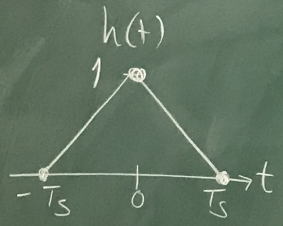
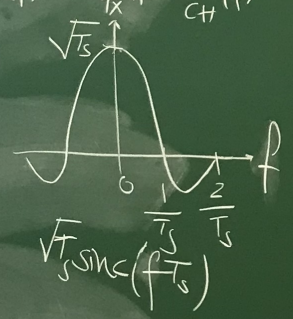
Ex:

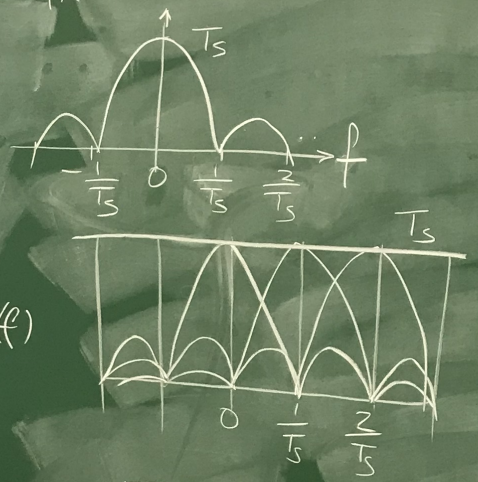




$H(f) = G_{TX}(f) \cdot \overbrace{G_{CH}(f)}^{1} \cdot G_{RX}(f)$



$H(f) = T_s \,\mathrm{sinc}^2(fT_s)$

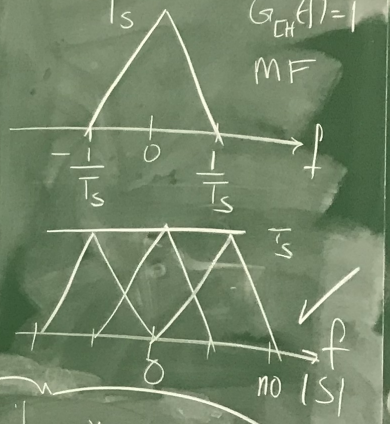


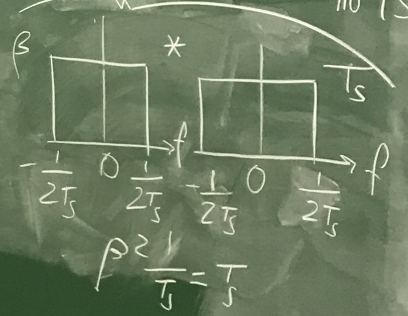
$\sum_k H\left(f - \frac{k}{T_s}\right) \overset{?}{=} T_s$

Ex: $H(f) = G_{TX}(f) G_{RX}(f)$

$G_{CH}(f) = 1$

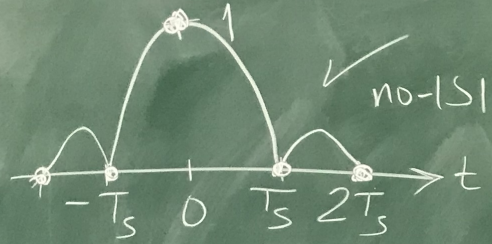
MF



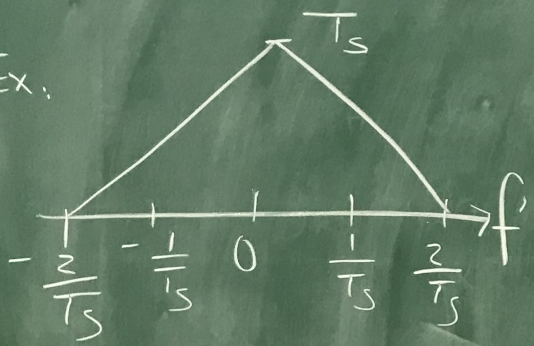


$\beta \geq \frac{1}{T_s} = T_s$

$h(t) = \mathrm{sinc}^2\left(\frac{t}{T_s}\right)$



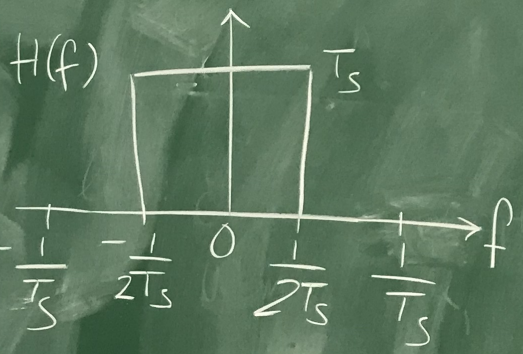
Ex.



Satisfy Nyquist?
Show in both time and freq.

Obtain H(f) that has minimum BW.

$$\sum H\left(f - \frac{k}{T_s}\right) = T_s$$



min BW soln