Carleton University

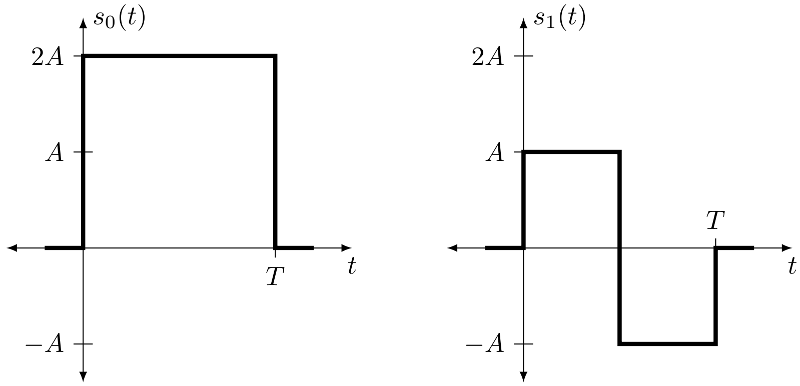
SYSC 5504 **Department of Systems and Computer Engineering**

ELG 6154 **Principles of Digital Communications** **Fall 2017/18**

Assignment #2

Due on Tuesday, November 14th, 2017

1. Consider a binary baseband communication system in which 0's and 1's are equally likely to be transmitted over and additive white Gaussian noise (AWGN) channel with single-sided noise power spectral density $\mathcal{N}_0$. The signals $s_0(t)$ and $s_1(t)$ shown below are used to convey a 0 and a 1, respectively.



   (a) Find a set of basis signals for $s_0(t)$ and $s_1(t)$, and express $s_0(t)$ and $s_1(t)$ in terms of these basis signals.

   (b) Plot $s_0(t)$ and $s_1(t)$ in a signal space diagram. Clearly indicate the decision regions used by a MAP receiver.

   (c) Find $\mathcal{E}_b$, the average transmitted energy per bit.

   (d) Find the probability of error for this communication system. Express your answer in terms of $\mathcal{E}_b$.

2. Consider a binary communication system where the decision device has two inputs, R_1 and R_2, given by

$$R_1 = 2S + W_1$$

$$R_2 = S + 2W_2$$

where W_1 and W_2 are statistically independent Gaussian random variables, each with zero mean and variance σ_W^2. The transmitted signal, S, is given by:

$$S = \begin{cases} A, & \text{if a 0 was transmitted} \\ -A, & \text{if a 1 was transmitted} \end{cases},$$

and is statistically independent of the noise components (W_1 and W_2). Assume that 0's and 1's are equally likely to be transmitted.

   (a) What rule should the decision device use to minimize the probability of a bit error? Express the rule in the simplest form possible.

   (b) What is the probability of a bit error if this decision rule is used? Express your answer in terms of A and σ_W^2.

3. Suppose the input to the decision device at the receiver of a binary communication system is modelled as

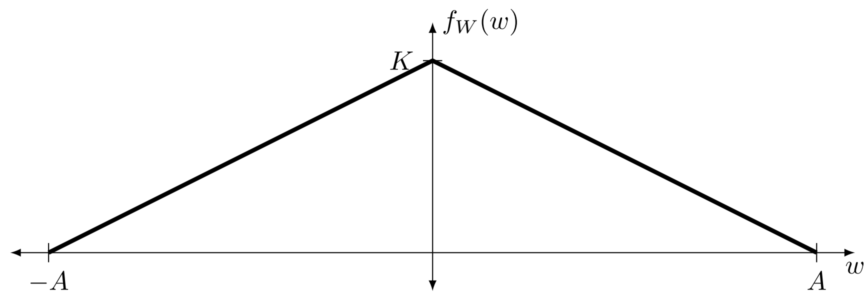
$$R = \begin{cases} 0 + W, & \text{if a 0 was transmitted} \\ A + W, & \text{if a 1 was transmitted} \end{cases},$$

where the noise variable, W, is characterized by the pdf shown below. You may not assume that 0's and 1's are equally likely to be sent, but instead let $\rho > \frac{1}{2}$ be the *a priori* probability that a 0 is sent.

(a) Express K in terms of A so that $f_W(w)$ is a valid pdf.

(b) Derive the optimal decision rule to minimize the probability of an error. Express the rule in the simplest form possible.

(c) What is the probability of a bit error if this decision rule is used? Express your answer in terms of A and ρ.



4. In the communication system shown below, the decision device has two inputs, R_1 and R_2, given by

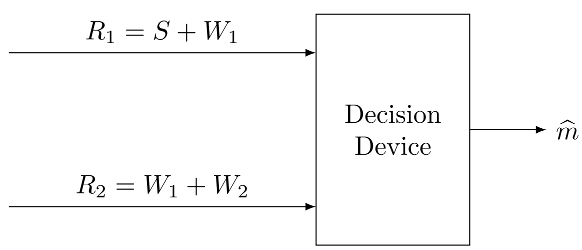
$$R_1 = S + W_1$$

$$R_2 = W_1 + W_2$$

where W_1 and W_2 are statistically independent Gaussian random variables, each with zero mean and variance σ_W^2. The transmitted signal, S, is given by:

$$S = \begin{cases} \sqrt{\mathcal{E}_b}, & \text{if a 0 was transmitted} \\ -\sqrt{\mathcal{E}_b}, & \text{if a 1 was transmitted} \end{cases},$$

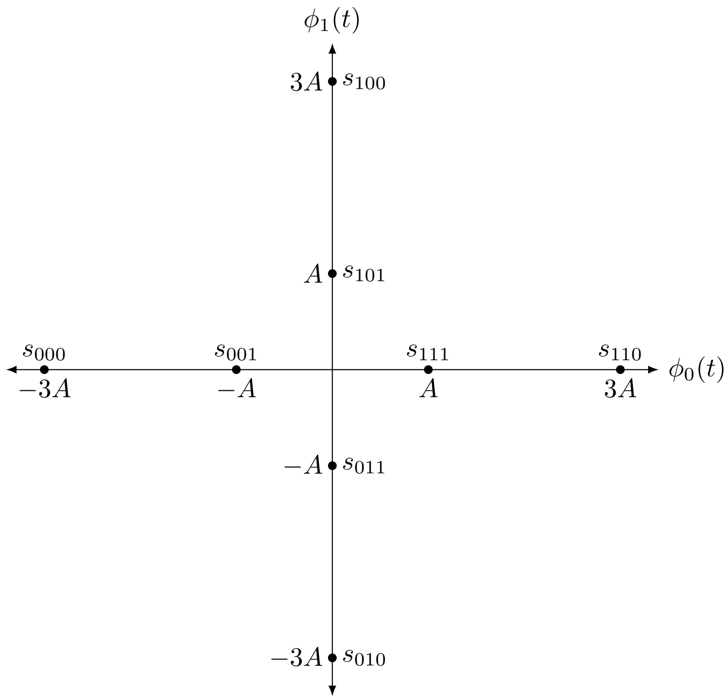
and is statistically independent of the noise components (W_1 and W_2). Assume that 0's and 1's are equally likely to be transmitted.



(a) What rule should the decision device use to minimize the probability of a bit error? Express the rule in the simplest form possible.

(b) What is the probability of a bit error if this decision rule is used? Express your answer in terms of $\mathcal{E}_b$ and σ_W^2.

5. Suppose the following signalling scheme is used for transmission over an AWGN channel with single-sided noise power spectral density of $\mathcal{N}_0$.



(a) Carefully draw the boundaries of the optimal decision regions in the signal space diagram. Be precise.

(b) Use the union bound to find a *tight* upper bound the probability of a symbol error. Express your answer in terms of $\mathcal{N}_0$ and the average transmitted energy per symbol ($\mathcal{E}_s$).