Laboratory #4

Probability of Error for BPSK and 2-PAM

SYSC 4600 Digital Communications

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Faculty of Engineering
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Purpose and Objectives

The purpose of this laboratory experiment is to investigate the probability of bit error for different digital bandpass modulation schemes. It builds on Laboratories #2 and #3, which you should review. You are encouraged to refer back to those lab manuals while working on this lab.

Background

One reason for running a computer simulation of a digital communication system is to estimate the system reliability in terms of the probability of error. Although computer simulation does not replace the need for field trials with actual hardware, it is nonetheless a useful tool. For example, the trouble and expense of implementing a system in hardware can be avoided if it can be shown through simulation that the system will not achieve acceptable performance. In addition, computer simulation can be used to compare the performance of different systems under operating conditions that may be difficult to create in the field.

For a given communication system operating in a specific environment, a computer simulation can provide an estimate of the probability of a bit error (also know as the bit error rate, or BER). Suppose you simulate the transmission of N_b message bits, and discover that N_ε of those bits are received incorrectly. The probability of a bit error can be estimated as

$$\hat{p} = \frac{N_\varepsilon}{N_b} \ .$$

It is important to remember that $\hat{p}$ is only an estimate of the actual probability of bit error for the system, p. In fact, if we repeated the experiment several times, it is very likely we would get different estimates of the probability of error each time. It is necessary to have some knowledge of the accuracy of the estimate. One useful measure of the accuracy is the *confidence interval*, $[d_L, d_H]$, such that $\Pr\{d_L \le p \le d_H\} = \beta$, where d_L and d_H are functions of $\hat{p}$ and β is the *confidence level*. Typical values of β are 90%, 95%, and 99%. For example, suppose that $\hat{p} = 5 \times 10^{-5}$ was found by simulation and used to construct a confidence interval of

$$\Pr\{4 \times 10^{-5} \le p \le 6 \times 10^{-5}\} = 0.95 \ .$$

This means there is a 95% chance that the true value of p falls between 4×10^{-5} and 6×10^{-5}.

After some mathematical manipulation, it is not difficult to show that the confidence interval for a confidence level of β is well approximated by

$$d_L = \frac{N_b}{N_b + \gamma^2}\left[\hat{p} + \frac{\gamma^2}{2N_b} - \gamma\sqrt{\frac{\hat{p}(1-\hat{p})}{N_b} + \frac{\gamma^2}{4N_b^2}}\right]$$

$$d_H = \frac{N_b}{N_b + \gamma^2}\left[\hat{p} + \frac{\gamma^2}{2N_b} + \gamma\sqrt{\frac{\hat{p}(1-\hat{p})}{N_b} + \frac{\gamma^2}{4N_b^2}}\right]$$

where $\gamma = \sqrt{2}\,\mathrm{erf}^{-1}(\beta)$.

Prelaboratory Exercise

1. Review labs #2 and #3.

Suppose BPSK with the rectangular pulse shape shown in Figure 1 is used for transmission over an AWGN channel with a double-sided noise PSD of $N_0/2$. The transmitted signals are $s_0(t) = h_T(t)\sqrt{2}\cos 2\pi f_c t$ if a 0 is sent, and $s_1(t) = -h_T(t)\sqrt{2}\cos 2\pi f_c t$ if a 1 is sent. Assume that the carrier frequency, f_c, is an integer multiple of the symbol rate, $1/T$.

2. Draw the signal space diagram.

3. Find the average transmitted energy per bit, E_b.

4. Find a simple mathematical expression for the probability of a bit error of an optimal receiver. Express your answer in terms of E_b and N_0.

5. Evaluate the expression you obtained in (4) for a signal-to-noise ratio of $E_b / N_0 = 5$ dB.

6. Repeat parts (2)-(5) for the 2-PAM modulation scheme, where $s_0(t) = 0$ and
   $s_1(t) = \sqrt{2}h_T(t)\sqrt{2}\cos 2\pi f_c t$.

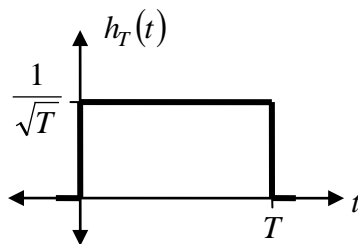
Figure 1. Normalized rectangular pulse shape.

Laboratory

Step 1: BER of BPSK

To estimate the probability of a bit error for BPSK, modify your simulator from Lab #2 by removing the instructions for calculating the PSD, and use the following system parameters:

Message word length	$N_a = 1000$ bits
Number of message words to test	$N_f = 10$ words
Symbol duration	$T = 0.01$ seconds
Sampling ratio	$\eta = 64$ samples per symbol
Carrier frequency	$f_c = 400$ Hz
Signal-to-noise ratio (SNR)	$E_b / N_0 = 5$ dB
Symbol Map	`SM = [+1 -1];`

Use a randomly-generated message. Keep track of the total number of message bit errors that occur over all N_f transmitted message words.

The AWGN bandpass channel model is

$$r_c(t) = v_c(t) + w_c(t)$$

where $v_c(t)$ is the transmitted bandpass signal, and $w_c(t)$ is the noise, which is modeled as a Gaussian random process with zero mean and a double-sided noise PSD of $N_0/2$. To simulate the bandpass noise in MATLAB, use

```
rct = vct + sqrt(1/Ts * N0/2) * randn(1, length(vct));
```

where `rct` is the received bandpass signal, `vct` is the transmitted bandpass signal, and `Ts` is the sampling period ($T_s = T/\eta$). The value for N_0 is calculated from $N_0 = E_b \times 10^{-SNRdB/10}$ where SNRdB is the desired SNR (E_b / N_0) in dB, and E_b is what you calculated in the pre-lab.

Question 1. Start by simulating $N_f = 10$ message words of $N_a = 1000$ bits each, at a signal-to-noise ratio of $E_b / N_0 = 5$ dB. What is the total number of bit errors that occurred? What is your estimate of the probability of a bit error? What are the upper and lower limits of the 95% confidence interval? What is the true probability of error as calculated using the equation of your pre-lab? How do these values compare?

Question 2. Run your simulator for different values of E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB. Record the number of errors and the estimated probability of bit error for each E_b/N_0 in a table.

Question 3. Use the results from Question 2 to generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the upper and lower limits of the 95% confidence interval, and the true probability of error.

Question 4. To get more accurate results, increase N_f to 1000. Rerun your simulator for E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB, and generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the upper and lower limits of the 95% confidence interval, and the true probability of error.

Step 2: Probability of Bit Error for 2-PAM

Modify your simulator to use the 2-PAM modulation scheme, where $s_0(t) = 0$ and $s_1(t) = \sqrt{2}h_T(t)\sqrt{2}\cos 2\pi f_c t$, and don't forget to update the decision rule. Using $N_f = 1000$, estimate the probability of bit error for E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB.

Question 5. Generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the upper and lower limits of the 95% confidence interval, and the true probability of error.

Laboratory Report Instructions

Please email a soft-copy of your .m file to your TA, and include a hard-copy with your report. Please email a soft-copy of your report to the TA in addition to submitting a hard-copy.