For a given $E$, best strategy is antipodal signaling. 

$$P_e = \frac{1}{2} \text{erfc} \left( \frac{d}{2 \sqrt{N_0}} \right) = \frac{1}{2} \text{erfc} \sqrt{\frac{E}{N_0}}$$

4-ary signaling in 2-D space: QPSK

45 degrees/qua

Model in Ch 4:

$$x = h(t) * x(t) \rightarrow \mathbf{w}(t)$$

L2: one tx filter

Model in Ch 5 and 6:

$m$-ary, $m$ different signal waveforms $\sum_{i=1}^{m} h(t, i)$. What type of receiver?

Decompose the received signal; i.e., project the received signal onto the signal space.

No noise:

With noise

$\mathbf{r}(t) = \mathbf{s}(t) + \mathbf{w}(t)$

The $N$-dimensional signal space is constructed to represent $m$ signal waveforms. White noise may have other dimensions. Actually, white noise has infinite dimensions (because it is a hypothetical signal with component power in all frequencies).
\[ r'(t) = r(t) - \sum_{j=1}^{N} r_j \phi_j(t) = r(t) - \sum_{j=1}^{N} (s_j + w_j) \phi_j(t) \]

(rejected part of the received signal by the receiver)

\[ = r(t) - \sum_{j=1}^{N} s_j \phi_j(t) - \sum_{j=1}^{N} w_j \phi_j(t) \]

\[ = s(t) + \omega(t) - s(t) - \sum_{j=1}^{N} w_j \phi_j(t) \]

\[ = \omega(t) - \omega'(t) \]

(noise projected on to the signal space)

(part of white noise outside the signal space -> that part of the noise is rejected)

what matters is the part of noise in the signal space.

\[ r'(t) \perp \text{signal space} \]

\[ r'(t) \perp s_i(t) \implies \text{does not matter} \]

\[ r'_j = s'_j + w'_j \]

\[ W_j = \int_{0}^{T} [w(t) \phi_j(t)] \, dt \]

\[ E[W_j] = 0 \]

\[ \sigma^2_{W_j} = E[W_j^2] = R_w(0) = \int_{-\infty}^{\infty} ||\phi(t)||^2 \, dt = \frac{N_0}{2} \]

\[ W(t) \xrightarrow{h(t)} W_j(t) \]

\[ S_{W_j}(f) = \int ||\phi(t)||^2 \, dt \]

\[ \phi_j(t) \]

\[ x(t) \eta(t) \]

\[ = \int_{0}^{T} \phi_j(t) \, dt \]

\[ \phi_j(t) \]

\[ x_j(t) \]

\[ \int_{0}^{T} \phi_j(t) \, dt \]

\[ y(t) = \int x(t) h(t-T) \, dt \]

\[ y(T) = \int x(t) h(T-T) \, dt \]

\[ h(T) = \phi_j(T-T) \]

\[ h(-T) = \phi_j(T+T) \]

\[ h(-\tau) = \phi_j(T+(-\tau-T)) \]

\[ h(T+\tau) = \phi_j(\tau) \]

\[ y(t) = \int x(t) \phi_j(t) \, dt \]

\[ = \int x(t) \phi_j(t) \, dt \]

\[ = x_j(t) \]

\[ x(t) \eta(t) \]
\[ \mathbf{G} = \mathbf{S}^i + \mathbf{W}^j \quad \Rightarrow \quad G(0; \sigma^2 = \frac{N_0}{2}) \]

\[ G(S_{ij}; \sigma^2 = \frac{N_0}{2}) \]

\[ \text{Cov} [R_j R_k] = E[(R_j - S_{ij})(R_k - S_{ik})] = E[W_j W_k] = E[W_j] E[W_k] = 0 \]

\[ R_j, R_k: \text{uncorrelated + Gaussian} \quad \rightarrow \quad \text{independent} \]

\[ f_R(\mathbf{r} | m_i) = \prod_{j=1}^{N} f_{R_j}(\mathbf{r}_j | m_i), \quad i = 1, \ldots, M \]

\[ \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(\mathbf{G} - S_{ij})^2}{2 N_0}} = \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(\mathbf{G} - S_{ij})^2}{N_0}} \]

\[ f_R(\mathbf{r} | m_i) = \frac{1}{(\pi N_0)^{N/2}} e^{-\frac{1}{N_0} \sum_{j=1}^{N} (\mathbf{G}_j - S_{ij})^2} \]

\[ \text{characterization of the observation vector } \mathbf{r} \text{ given } m_i \text{ is transmitted.} \]

5.4 Likelihood Function

\[ L(m_i) = f_R(\mathbf{r} | m_i) \]

\[ \text{LLR: log-likelihood function} \quad \Lambda(m_i) = \log L(m_i) = -\frac{1}{N_0} \sum_{j=1}^{N} (\mathbf{G}_j - S_{ij})^2 \]

5.5 Coherent Detection of Signals in Noise: Maximum Likelihood Decoding

\[ P_e(\hat{m} = m_i) = P(m_i \text{ not sent} | \mathbf{r}) = 1 - P(m_i \text{ sent} | \mathbf{r}) \]

Optimum decision rule:

\[ \begin{align*}
\text{Set } \hat{m} & = m_i \text{ if } \\
\text{Set } \hat{m} & = m_i \text{ if } P(m_i \text{ sent} | \mathbf{r}) > P(m_k \text{ sent} | \mathbf{r}) \quad \forall k \neq i \\
\text{Set } \hat{m} & = m_i \text{ if } P(m_k \text{ sent} | \mathbf{r}) \text{ is maximum for } k \neq i \\
\text{Set } \hat{m} & = m_i \text{ if } P(m_k \text{ sent} | \mathbf{r}) \text{ is maximum for } k \neq i \\
\text{Set } \hat{m} & = m_i \text{ if } maximum \text{ a posteriori probability (MAP rule)}
\end{align*} \]
if symbols are equally likely

\[ \text{Set } \hat{m} = m_i \text{ if } \sum_{j=1}^{N} (r_j - s_{kj})^2 \text{ is minimum for } k=i \]

minimum distance (MD) rule

\[ \text{Set } \hat{m} = m_i \text{ if } \frac{1}{N} \sum_{j=1}^{N} (r_j - s_{kj})^2 \text{ is maximum for } k=i \]

maximum likelihood (ML) rule

\[ L(m_k) = -\frac{1}{N} \sum_{j=1}^{N} (r_j - s_{kj})^2 \]

5.6 Correlator Receiver

\[ \text{set } \hat{m} = m_i \text{ if } \sum_{j=1}^{N} (r_j - s_{kj}) = \frac{1}{2} E_k \text{ is maximum for } k=i \]

5.6 Correlator Receiver

\[ \frac{r_{m_k}}{E_k} = \frac{r_{m_k}}{E_k} \]

correlator

receiver

[Diagram of a correlator receiver]
5.7 Probability of Error

\[ P_e = \sum_{i=1}^{M} P_{e|c_i} = \frac{1}{M} \sum_{i=1}^{M} P(\bar{c} \text{ not in } z_i | m_i) = 1 - \frac{1}{M} \sum_{i=1}^{M} P(\bar{c} \text{ in } z_i | m_i) \]

average probability of symbol error

\[ = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{z_i} p_c(\bar{c}|m_i) \, d\bar{c} \]

\[ \text{Same error performance} \]

-3x \ -x \ 0 \ x \ 3x