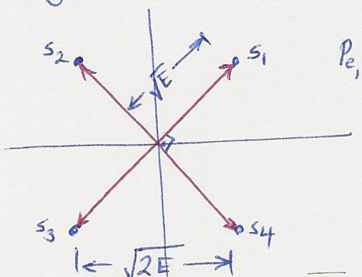
For a given E, best strategy is antipodal signaling.

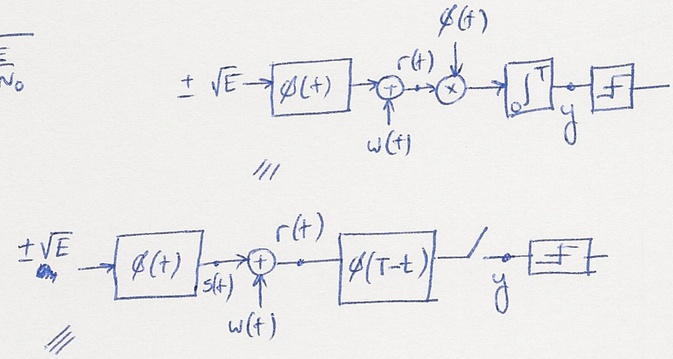
$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d}{2\sqrt{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E}{N_0}}$$

4-ary signaling in 2-D space : QPSK

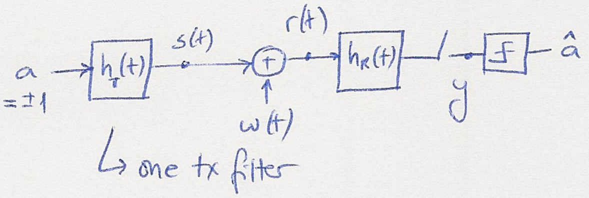


$$P_{e,1\to 2} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E}{2N_0}}$$

Oct 9/2008



Model in Ch 4 :

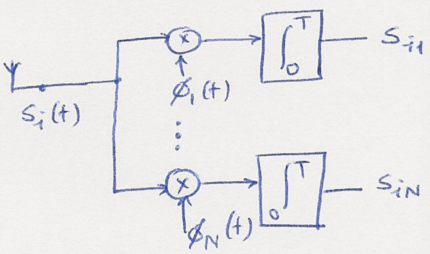


binary with one waveform

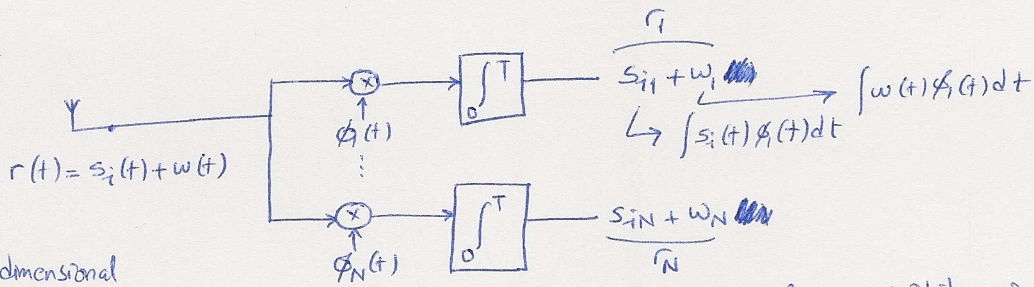
Model in Ch 5 and 6: M-ary, M different signal waveforms $\{h_{T,i}(t)\}_{i=1}^{M}$. What type of receiver?

Decompose the received signal; i.e., project the received signal onto the signal space.

No noise :



With noise



Note: The N-dimensional signal space is constructed to represent M signal waveforms. White noise may have other dimensions. Actually white noise has infinite dimensions, (because it is a hypothetical signal with ~~componen~~ power in all frequencies).

$$r'(t) = r(t) - \sum_{j=1}^{N} r_j \phi_j(t) = r(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\, \phi_j(t)$$

rejected part of the received signal by the receiver

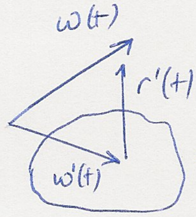
$$= r(t) - \sum_{j=1}^{N} s_{ij} \phi_j(t) - \sum_{j=1}^{N} w_j \phi_j(t)$$

$$= s_i(t) + w(t) - s_i(t) - \underbrace{\sum_{j=1}^{N} w_j \phi_j(t)}_{\text{noise projected on to the signal space}}$$

$$= \underbrace{w(t) - w'(t)}_{}$$

part of white noise outside the signal space → that part of the noise is rejected

→ what matters is the part of noise in the signal space.



$r'(t) \perp$ signal space

$r'(t) \perp s_i(t) \Rightarrow$ does not matter

$$R_j = S_{ij} + W_j \qquad W_j = \int_0^T W(t)\,\phi_j(t)\,dt \;\to \text{zero mean}$$

$$E[W_j] = 0 \qquad \sigma_{W_j}^2 = E[W_j^2] = R_W(0) = \int_{-\infty}^{\infty} \frac{N_0}{2} \|\Phi(f)\|^2 df = \frac{N_0}{2} \int \|\phi(t)\|^2 dt = \frac{N_0}{2}$$

$$W(t) \to \boxed{h(t)} \xrightarrow{W_j(t)} \llcorner \; W_j \qquad S_{W_j}(f) = \int |H(f)|^2 S_W(f)\, df$$

$$x(t) \to \otimes_{\phi_j(t)} \to \boxed{\int_0^T} \to x_j \;\equiv\; x(t) \to \boxed{h(t) = \phi_j(T-t)} \xrightarrow{y(t)} \underset{t=T}{\diagup} \to y(T)$$

$$y(t) = \int x(\tau) h(t-\tau)\, d\tau$$

$$y(T) = \int x(\tau) h(T-\tau)\, d\tau$$

$$h(\tau) = \phi_j(T-\tau)$$
$$h(-\tau) = \phi_j(T+\tau)$$
$$h(-[\tau - T]) = \phi_j(T + (\tau - T))$$
$$h(T-\tau) = \phi_j(\tau)$$

$$y(T) = \int x(\tau) \phi_j(\tau)\, d\tau$$
$$= \int x(t) \phi_j(t)\, dt$$
$$= x_j$$

$R_j = s_{ij} + W_j$

$\hookrightarrow G(0; \sigma^2 = \frac{N_0}{2})$

$\hookrightarrow G(s_{ij}; \sigma^2 = \frac{N_0}{2})$

$$\bar{R} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

$$\mathrm{Cov}[R_j R_k] = E\left[(R_j - s_{ij})(R_k - s_{ik})\right] = E[W_j W_k] = \underbrace{E[W_j]}_{0}\,\underbrace{E[W_k]}_{0} = 0$$

R_j, R_k : uncorrelated + Gaussian $\longrightarrow$ independent

$$f_{\bar{R}}(\bar{r}|m_i) = \prod_{j=1}^{N} \underbrace{f_{R_j}(r_j|m_i)}, \qquad i = 1, \dots, m$$

$$\frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(r_j - s_{ij})^2}{2\frac{N_0}{2}}} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r_j - s_{ij})^2}{N_0}}$$

$$f_{\bar{R}}(\bar{r}|m_i) = \frac{1}{(\pi N_0)^{N/2}} e^{-\frac{\sum_{j=1}^{N}(r_j - s_{ij})^2}{N_0}}$$

$\hookrightarrow$ characterization of the observation vector $\bar{r}$ given m_i is transmitted.

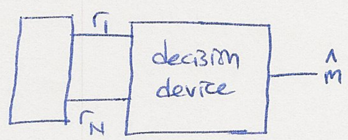
5.4 Likelihood Function

$L(m_i) = f_{\bar{R}}(\bar{r}|m_i)$

LLR: log-likelihood function $\quad l(m_i) = \log L(m_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2$

5.5 Coherent Detection of Signals in Noise: Maximum Likelihood Decoding

$$\underset{\hat{m} = m_i}{P_e}(m_i|\bar{r}) = P(m_i \text{ not sent}|\bar{r}) = 1 - P(m_i \text{ sent}|\bar{r})$$



Optimum decision rule:

$$\boxed{\begin{array}{c}\text{Set } \hat{m} = m_i \text{ if} \\ P(m_i \text{ sent}|\bar{r}) \geq P(m_k \text{ sent}|\bar{r})\end{array}} \quad \forall\, k \neq i$$

$P(m_k \text{ sent}|\bar{r})$ is maximum for $k = i$

Maximum a posteriori probability

(MAP rule)

if symbols are equally likely

$$\boxed{\text{set } \hat{m} = m_i \text{ if } f_{\bar{R}}(\bar{r} \mid m_k) \text{ is maximum for } k = i}$$

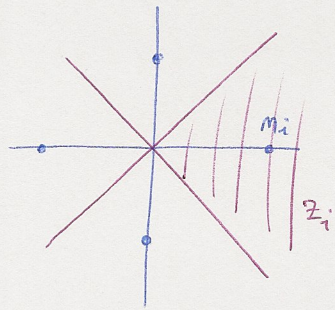
maximum likelihood (ML) rule

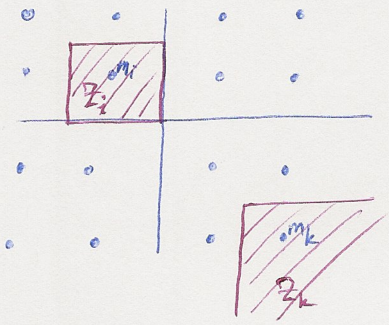
$$\boxed{\ell(m_k) = -\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{kj})^2 \text{ is maximum for } k = i}$$

$$\boxed{\text{set } \hat{m} = m_i \text{ if } \sum_{j=1}^{N} (r_j - s_{kj})^2 = \|r - s_k\|^2 \text{ is minimum for } k = i}$$

minimum distance (MD) rule

$$\boxed{\text{set } \hat{m} = m_i \text{ if } \bar{r} \text{ is in } Z_i}$$



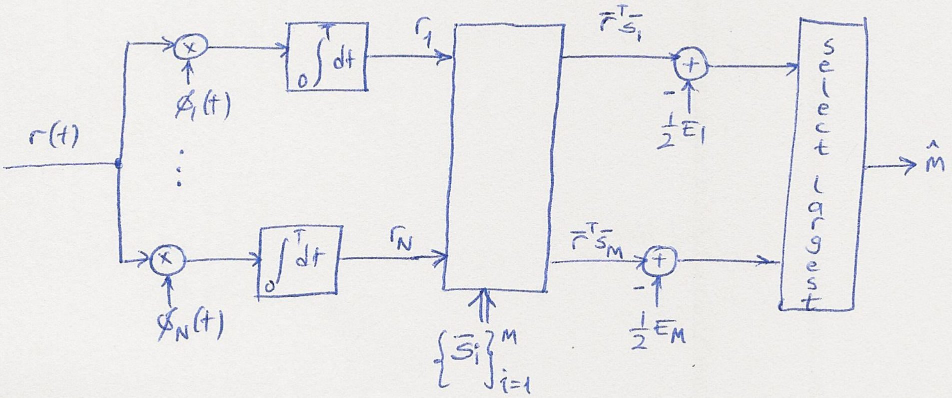


$$\sum_{j=1}^{N} (r_j - s_{kj})^2 = \sum r_j^2 - 2 \underbrace{\sum r_j s_{kj}}_{\bar{r} \cdot \bar{s}_k} + \underbrace{\sum s_{kj}^2}_{E_k}$$

$$\boxed{\text{set } \hat{m} = m_i \text{ if } \underbrace{\sum_{j=1}^{N} r_j s_{kj}}_{\bar{r} \cdot \bar{s}_k = \bar{r}^T \bar{s}_k} - \frac{1}{2} E_k \text{ is maximum for } k = i}$$

correlator receiver

5.6 Correlator Receiver



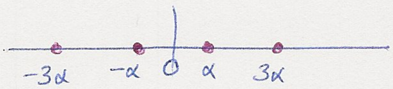
5.7 Probability of Error

equally-likely assumption

$$P_e = \sum_{i=1}^{M} P_{e|i}\, P_i = \frac{1}{M}\sum_{i=1}^{M} P(\bar{r} \text{ not in } Z_i \mid m_i) = 1 - \frac{1}{M}\sum P(\bar{r} \text{ in } Z_i \mid m_i)$$

average probability of symbol error

$$= 1 - \frac{1}{M}\sum_{i=1}^{M} \int_{Z_i} f_{\bar{R}}(\bar{r} \mid m_i)\, d\bar{r}$$



same error performance