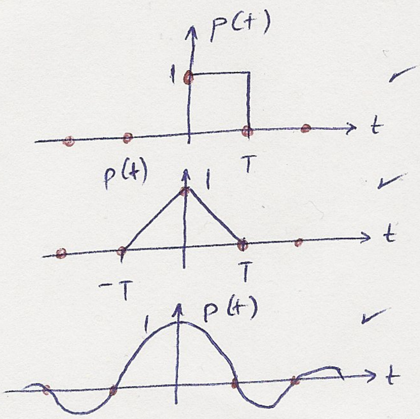
may or may not be a MF

$$\sum a_k \delta(t-kT) \to \boxed{h_T(t)} \to \boxed{h_c(t)} \to \oplus \to \boxed{h_R(t)} \to \hat{a}_k$$

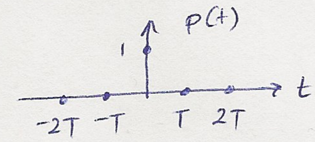
$w(t)$

Omit noise $\quad p(t) = h_T(t) * h_c(t) * h_R(t)$

$$\sum a_k \delta(t-kT) \to \boxed{p(t)} \to \hat{a}_k$$



no ISI condition in time domain



$$p(t) = \begin{cases} 1, & t=0 \\ 0, & t=kT \end{cases}$$

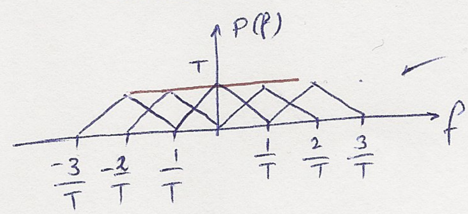
Find the no ISI cond in freq domain

$$p(t) \sum_{k=-\infty}^{\infty} \delta(t-kT) = \delta(t)$$

$$P(f) * \frac{1}{T} \sum \left(f - \frac{k}{T}\right) = 1$$

$$\boxed{\sum P\left(f - \frac{k}{T}\right) = T}$$

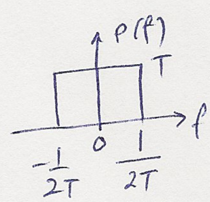
consider a $\text{sinc}^2\left(\frac{t}{T}\right) \to$ triangular (f)

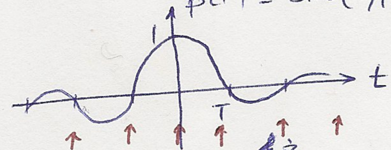


infinitely many solutions.

Find the minimum BW solution.

By inspection





$p(t) = \text{sinc}(t/T)$

spectral efficiency: in order to transmit at rate $R \left(= \frac{1}{T}\right)$, how much Bandwidth needed? $\frac{R}{2}$

Baseband: $\eta_{max} = 2$ sym/sec/Hz

$\boxed{\text{Bandpass: } \eta_{max} = 1 \text{ sym/sec/Hz}}$

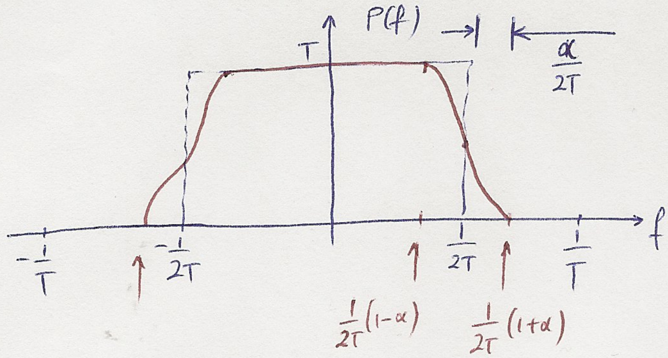
— Sep 30

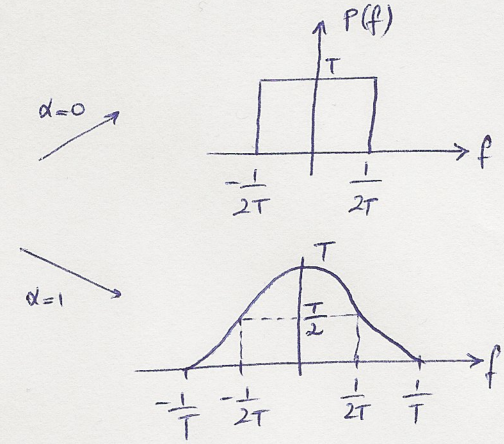
Time-domain sinc pulses cause practical problems. For instance, not robust wrt timing (synchronization) errors.

Raised cosine pulses

$$p(t) = \text{sinc}\left(\frac{t}{T}\right)\left(\frac{\cos \frac{\pi \alpha t}{T}}{1 - \frac{4\alpha^2 t^2}{T^2}}\right)$$

$$P(f) = \begin{cases} T, & 0 \le |f| < f_1 \\ \frac{T}{2}\left\{1 - \sin\left(\frac{\pi\left(|f| - \frac{1}{2T}\right)}{\frac{1}{T} - 2f_1}\right)\right\}, & f_1 \le |f| < \frac{1}{T} - f_1 \\ 0, & |f| \ge \frac{1}{T} - f_1 \end{cases}$$





$100\alpha\%$: excess BW

Spectral Efficiency: $\frac{1}{1+\alpha}$ sym/sec/Hz — Bandpass

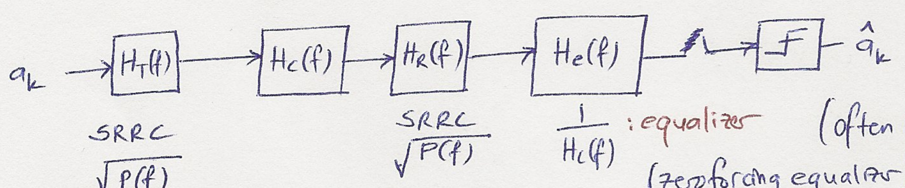
$\frac{2}{1+\alpha}$ sym/sec/Hz — Baseband

Note that we have been operating on $P(f) = H_T(f)\,H_c(f)\,H_R(f)$

Consider $H_c(f) = \delta(f)$

$H_R(f)$ and $H_T(f)$ must be matched filters to have the best error performance.

$|H_T(f)| = |H_R(f)| = \sqrt{P(f)}$ square-root raised-coise filters (SRRC)



(often implemented in discreet domain)

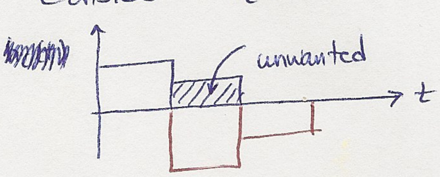
(zeroforcing equalizer)

Read pp [redacted], Haykin

Eye Patterns

$\boxed{\text{Equalization}}$ (pp [redacted], Haykin)

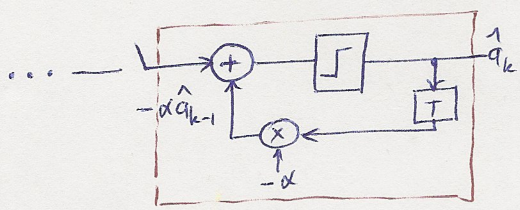
Consider $h_c(t) = \delta(t) + \alpha(t-T)$

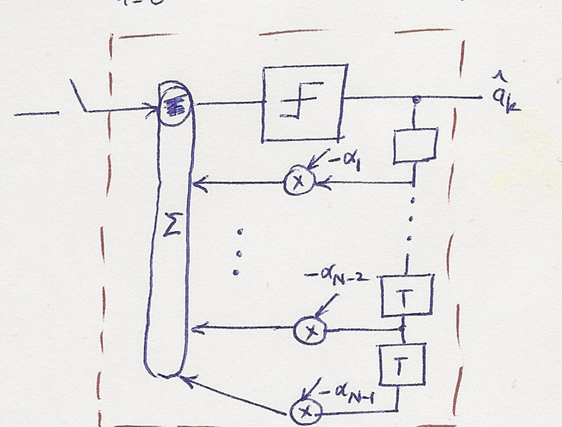


received signal in the absence of noise

$h_c(t) = \sum_{i=0}^{N-1} \alpha_i\, \delta(t-iT)$ assume wlog $\alpha_0 = 1$

Decision Feeback Equalizer



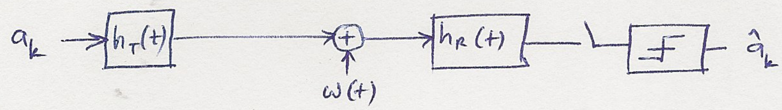


Case I) no channel (ideal channel) but noise

ideal channel: $h_c(t) = \alpha\,\delta(t-t_0)$

attenuation

propagation delay



optimal receiver: MF + threshold detector

$h_R(t) = h_T(T-t)$

$|H_R(f)| = |H_T(f)|$

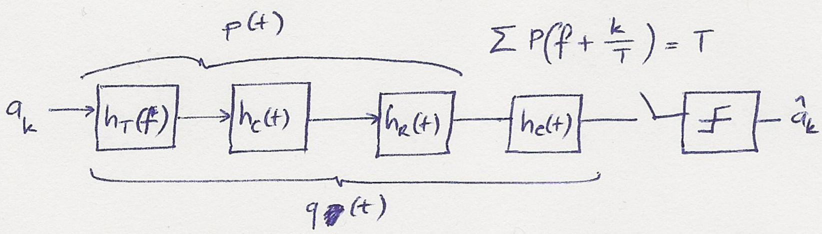
$BER = P_e = f\left(\mathrm{erfc}\left[f(\sqrt{E/N_0})\right]\right)$

BER may be made arbitrarily small, but $P_e \neq 0$

Case II) no noise but channel $\rightarrow$ derive no-ISI criterion for $P(f) = H_T(f)H_c(f)H_R(f)$

$$p(t) = \begin{cases} 1, & t=0 \\ 0, & t=kT \end{cases}$$

$$\sum P\left(f+\frac{k}{T}\right) = T$$



But $h_c(t)$ may be time varying $\rightarrow$ make $q(t)$ no-ISI by introducing $\overbrace{h_e(t)}^{\text{equalizer}}$.

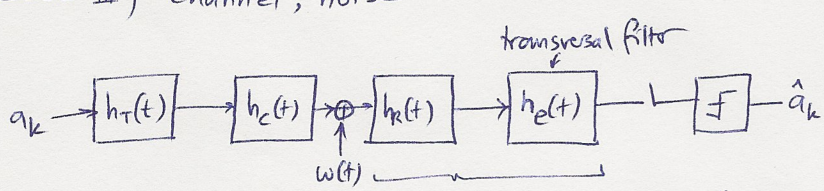
One realization $h_T(t), h_R(t)$: MF pair

$|H_T(f)| = |H_R(f)| = \sqrt{P(f)}$ SRRC filters

$|H_e(f)| = \dfrac{1}{|H_c(f)|}$ ... channel inversion (zero-forcing)

BER = 0

Case III) channel, noise



optimum linear receiver (p. 282-287, Haykin)

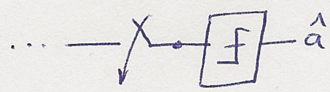
effective against both noise and ISI

minimizes mean-square error $\rightarrow$ MMSE receiver

Tail of a Gaussian PDF

$$P_e = P(\hat{a} \neq a) = \sum_i P_{e|i} P_i$$

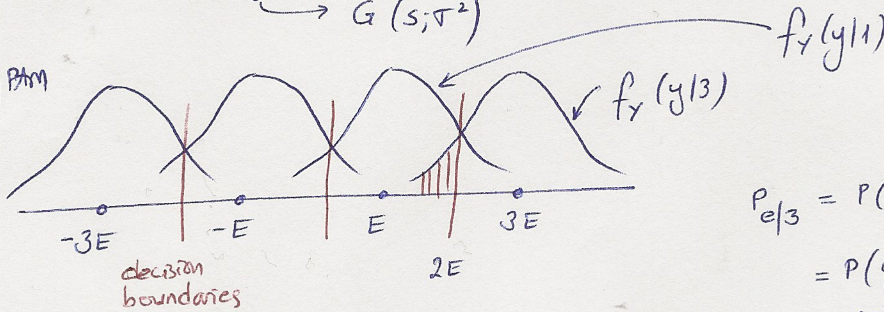
Rx:



$y = s + n \longrightarrow G(0;\sigma^2)$

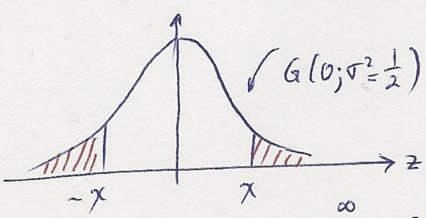
$\quad \longrightarrow G(s;\sigma^2)$

4-ary PAM

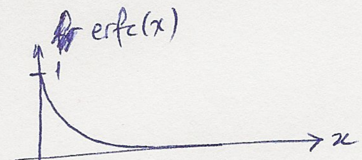


$$\begin{aligned} P_{e|3} &= P(\hat{a} \neq 3 \mid a=3) \\ &= P(y < 2E \mid a=3) \\ &= \int_{-\infty}^{2E} f_Y(y|3)\,dy = \int_{-\infty}^{2E} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y-3E)^2}{2\sigma^2}}\,dy \\ &= \int_{-\infty}^{-E} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y^2}{2\sigma^2}}\,dy = \int_{E}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y^2}{2\sigma^2}}\,dy \end{aligned}$$

Gaussian tail dies out very quickly $\longrightarrow P_e$: low



$$\text{Shaded Area} = 2 \times \frac{1}{\sqrt{2\pi}\,\frac{1}{\sqrt{2}}} \int_x^{\infty} e^{-\frac{z^2}{2\cdot\frac{1}{2}}}\,dz = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-z^2}\,dz \triangleq \mathrm{erfc}(x)$$



draw in log scale

Binary Antipodal PAM: $\quad P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E}{N_0}}$