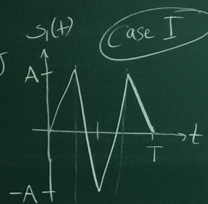
29 Nov 2016

Binary

$E_1 = E_2 = E$

Case I



$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d_{12}}{2\sqrt{N_0}}\right)}$$

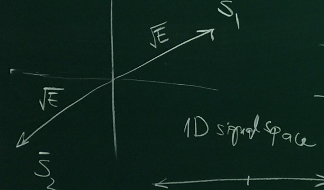
antipodal signalling

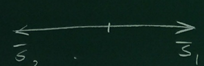
$\rightarrow P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E}{N_0}}$

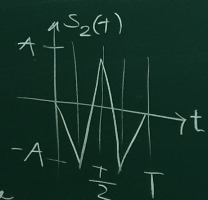
all equal-energy antipodal signalling schemes result in the same

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E}{N_0}}}$$

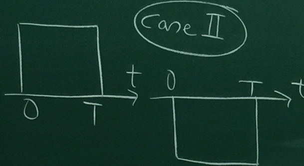
Design $s_2(t)$ that will result in minimum BER.





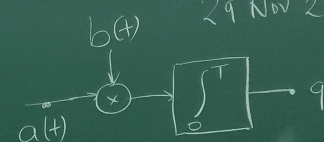


Case II



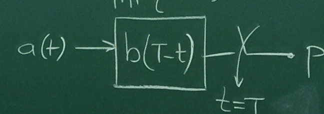
As long as $E_{I} = E_{II}$,

$P_{e,I} = P_{e,II}$



$\mathrm{MF}\{b(t)\}$

$q \overset{?}{=} p$



$p = a(t) * b(T-t)\Big|_{t=T}$

$\triangleq \int a(\tau)\, b(\tau)\, d\tau$

$= \int a(t)\, b(t)\, dt$

$= q$ ✓

$b(T-t) \rightarrow b(T-\tau) \rightarrow b(T+\tau) \rightarrow b(T+[\tau-t])$

$x(t) \rightarrow x(\tau) \rightarrow x(-\tau) \rightarrow x(-[\tau-t])$

$= x(t-\tau)$

Correlation receiver

= inner product receiver

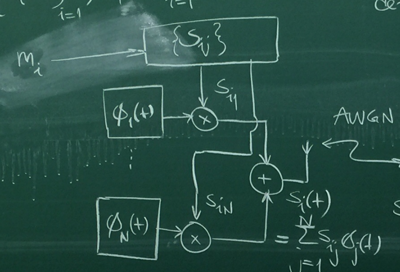
= matched filter receiver

signal-space analysis:

* systematic
* structured
* scalable
* normalized

M-ary signalling

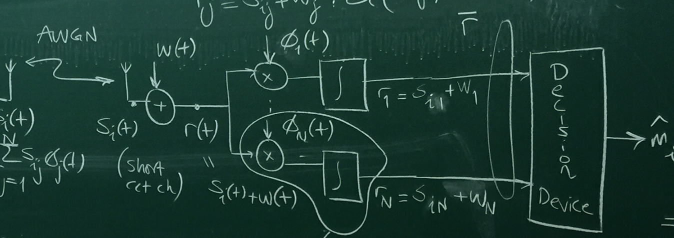
$\{s_i(t)\}_{i=1}^{M}, \quad \{\phi(t)\}_{i=1}^{N}$



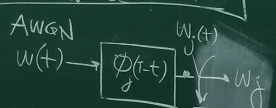
$r_1 = \underbrace{s_{i1}}_{\text{deterministic}} + \underbrace{w_1}_{\text{random}} : RV$

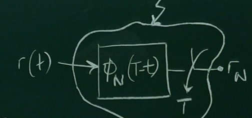
$w_j : G\left(0; \sigma^2 = \frac{N_0}{2}\right)$

$r_j = s_{ij} + w_j : G\left(s_{ij}; \sigma^2 = \frac{N_0}{2}\right)$



AWGN





$w_j = \int w(t)\phi_j(t)\,dt$

Since $w(t)$: Gaussian

$\rightarrow w_j$: Gaussian RV

$E[w_j] = E\left[\int w(t)\phi_j(t)\,dt\right]$

$= \int \underbrace{E[w(t)]}_{0} \phi_j(t)\,dt$

$= 0$

$\sigma_{w_j}^2 = E[w_j^2] - \cancel{E^2[w_j]}^{\,0} = P_{w_j} = \int_{-\infty}^{\infty} S_{w_j}(f)\,df = \int S_w(f)\,|\phi_j(f)|^2\,df = \frac{N_0}{2}\int |\phi_j(f)|^2\,df$

$= \frac{N_0}{2}\int |\phi_j(t)|^2\,dt$

$= \frac{N_0}{2}$

$r_1, r_2, \ldots, r_N \rightarrow$ Decision Device $\rightarrow \hat{m}$, $\{m_1, \ldots, m_M\}$

$r_j = s_{ij} + w_j$

$r_j : G\left(s_{ij}, \frac{N_0}{2}\right)$

$$f_{R_j}(r_j) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(r_j - s_{ij})^2}{2\frac{N_0}{2}}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r_j - s_{ij})^2}{N_0}}$$

Verify that $\{r_j\}_{j=1}^{N}$ are independent

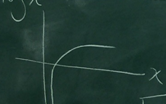
$$f_R(\bar{r}) = \prod_{j=1}^{N} f_{R_j}(r_j)$$

$$= \prod_{j=1}^{N} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r_j - s_{ij})^2}{N_0}}$$

$$= \frac{1}{(\pi N_0)^{N/2}} e^{-\frac{\sum (r_j - s_{ij})^2}{N_0}}$$

$\boxed{\text{Likelihood function}}$ captures everything about my observation in a statistical manner

log x



LL : log-likelihood function

$$\boxed{\log\left[f_R(\bar{r})\right] = -\frac{\sum (r_j - s_{ij})^2}{N_0}}$$

Given $\bar{r}$: Observation vector, find the decision rule that will minimize P_e.

Assume m_i is transmitted

minimize $P_e = P(\hat{m} \neq m_i \mid \bar{r})$

$= 1 - P_{correct} = 1 - P(\hat{m} = m_i \mid \bar{r})$

$$\boxed{\hat{m} = \arg\max_i P(m_i \mid \bar{r})}$$

maximum a posteriori principle (MAP)

Compute $P(m_1 \mid \bar{r}), \ldots, P(m_M \mid \bar{r})$

Choose the one that is highest

Note : $\sum_{i=1}^{M} P(m_i \mid \bar{r}) = 1$

THE BEST RECEIVER