Nov 22/2016

Given $\left\{ \bar{x}_i \right\}_{i=1}^{M}$, construct a signal space

* Inspection
* G-S Orthonalization

$\left\{ \bar{u}_i \right\}_{i=1}^{N}$, $N < M$

Most commonly, $N \ll M$

$$\bar{x}_i = \sum_{j=1}^{N} x_{ij} \, \bar{u}_j \qquad x_{ij} \rightarrow \bar{x}_i \cdot \bar{u}_j$$

① $\bar{u}_1 = \dfrac{\bar{x}_1}{\|\bar{x}_1\|}$

② $\bar{w}_2 = \bar{x}_2 - \overbrace{\bar{x}_2 \cdot \bar{u}_1}^{\text{scalar}} \, \bar{u}_1$

$\bar{u}_2 = \dfrac{\bar{w}_2}{\|\bar{w}_2\|}$

③ $\bar{w}_i = \bar{x}_i - \sum_{j=1}^{i-1} \bar{x}_i \cdot \bar{u}_j \, \bar{u}_j$

$\bar{u}_i = \dfrac{\bar{w}_i}{\|\bar{w}_i\|}$

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_M \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1N} \\ & \vdots & \\ x_{M1} & \cdots & x_{MN} \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \vdots \\ \bar{u}_N \end{bmatrix}$$

$\left\{ s_i(t) \right\}_{i=1}^{M}$ $\qquad$ $\left\{ \phi_i(t) \right\}_{i=1}^{N}$

① $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_{s_1}}}$

② $w_2(t) = s_2(t) - \overbrace{\left(s_2(t), \phi_1(t) \right)}^{\text{scalar}} \phi_1(t)$

$\phi_2(t) = \dfrac{w_2(t)}{\sqrt{E_{w_2}}}$

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \, \phi_j(t) \qquad s_{ij} \rightarrow \left(s_i(t), \phi_j(t) \right)$$

③ $w_i(t) = s_i(t) - \sum_{j=1}^{i-1} \left(s_i(t), \phi_j(t) \right) \phi_j(t)$

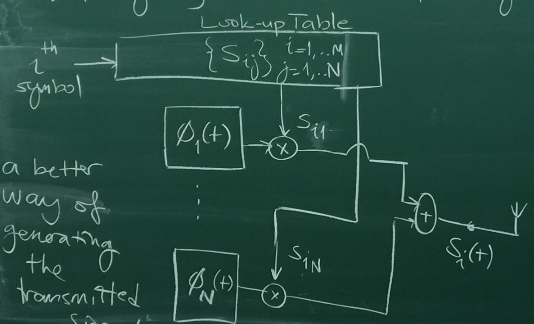
$\phi_i(t) = \dfrac{w_i(t)}{\sqrt{E_{w_i}}}$

$$\begin{bmatrix} s_1(t) \\ \vdots \\ s_M(t) \end{bmatrix} = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ & \vdots & \\ s_{M1} & \cdots & s_{MN} \end{bmatrix} \begin{bmatrix} \phi_1(t) \\ \vdots \\ \phi_N(t) \end{bmatrix}$$

M-ary signalling $\longrightarrow \log_2 M$ bits/sym

* Brute-force generation — Ex: 4096-ary

$\rightarrow$ M signal generator — 12 bits/sym



a better way of generating the transmitted signal

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t, \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t, \quad 0 \le t \le T$$

M-PSK

$$S_i(t) = A\cos(2\pi f_c t + \theta_i), \quad \theta_i = \frac{2\pi}{M}(i-1), \quad i=1,\dots,M \qquad 0 \le t \le T$$

$$= \underbrace{A\cos\theta_i}_{\text{scalar}} \cos 2\pi f_c t - \underbrace{A\sin\theta_i}_{\text{scalar}} \sin 2\pi f_c t$$

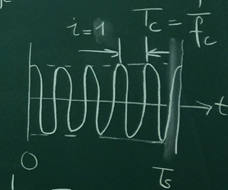
$(\sin 2\pi f_c t, \cos 2\pi f_c t)$

$$\triangleq \int_0^T \sin 2\pi f_c t \cos 2\pi f_c t \, dt$$

$$= \int_0^T \frac{1}{2} \sin 2\cdot 2\pi f_c t \, dt = 0$$

$\rightarrow$ sin and cos : $\perp$

$$E_{\sin} = \int_0^T \cos^2 2\pi f_c t \, dt = \int_0^T \frac{1}{2}(1 + \cos 2\cdot 2\pi f_c t)\, dt = \frac{T}{2}$$



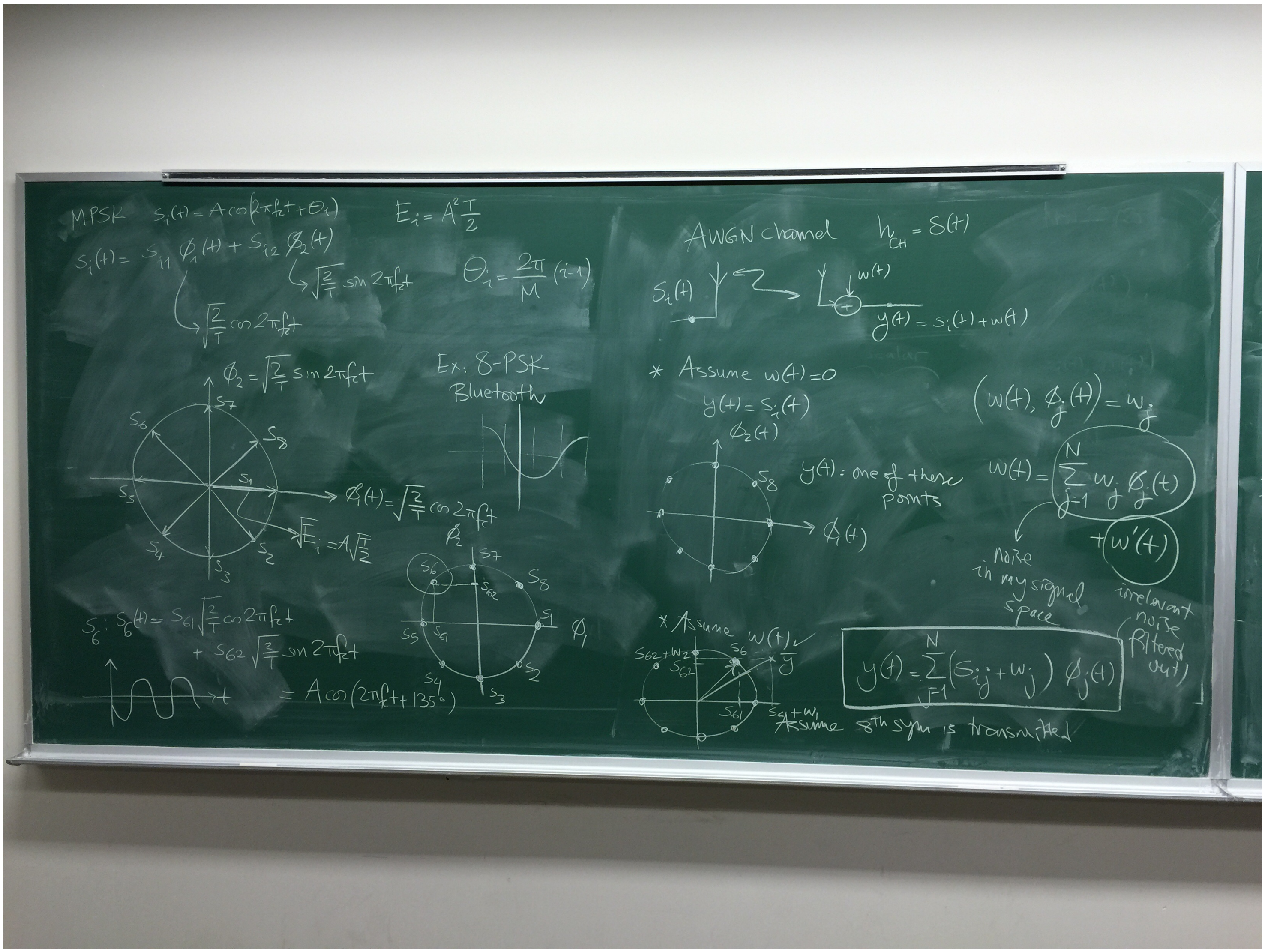
$$\frac{1}{R_s} = T_s \gg T_c = \frac{1}{f_c}$$

$$R_s \ll f_c$$

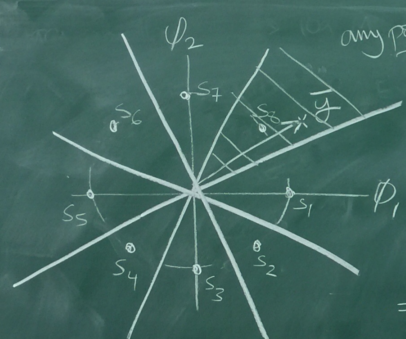
Ex: $R_s = 1M$ sym/sec

$f_c = 1G$ Hz

Assume $T_s = n T_c$ ↳ integer



MPSK $s_i(t) = A\cos(2\pi f_c t + \theta_i)$
$E_i = A^2 \frac{T}{2}$
$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$
$\sqrt{\frac{2}{T}} \sin 2\pi f_c t$
$\sqrt{\frac{2}{T}} \cos 2\pi f_c t$
$\theta_i = \frac{2\pi}{M}(i-1)$
$\phi_2 = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$
Ex: 8-PSK
Bluetooth
$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$
$\sqrt{E_i} = A\sqrt{\frac{T}{2}}$
s_6: $s_6(t) = s_{61}\sqrt{\frac{2}{T}} \cos 2\pi f_c t + s_{62}\sqrt{\frac{2}{T}} \sin 2\pi f_c t$
$= A\cos(2\pi f_c t + 135°)$
AWGN channel $h_{CH} = \delta(t)$
$s_i(t)$
$w(t)$
$y(t) = s_i(t) + w(t)$
* Assume $w(t) = 0$
$y(t) = s_i(t)$
$\phi_2(t)$
$y(t)$: one of these points
$\phi_1(t)$
$(w(t), \phi_j(t)) = w_j$
$w(t) = \sum_{j=1}^{N} w_j \phi_j(t) + w'(t)$
noise in my signal space
irrelevant noise (filtered out)
* Assume $w(t)$
$s_{62}+w_2$
$s_{61}+w_1$
Assume 6th sym is transmitted
$y(t) = \sum_{j=1}^{N} (s_{ij} + w_j)\,\phi_j(t)$



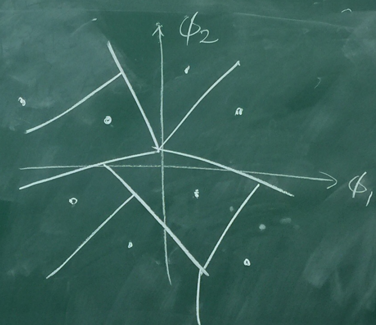
any point in this region is "closer" to s_8 than any other s_i

"close" = "resembling"

$\hat{i} = \arg\min_i \|\bar{y} - \bar{s}_i\|$ $\boxed{\text{optimal detection}}$

Maximum Likelihood (ML) detection [best bet]

= minimum distance (MD) detection (if all symbols are equally-likely)



8-ary $\quad P_i = \frac{1}{8}$

$$P_{e,s} = \left(P_{e|s_1} \times \frac{1}{8}\right) + \cdots + \left(P_{e|s_8} \times \frac{1}{8}\right)$$

$$= \frac{1}{M} \sum_{i=1}^{M} P_{e|i}$$