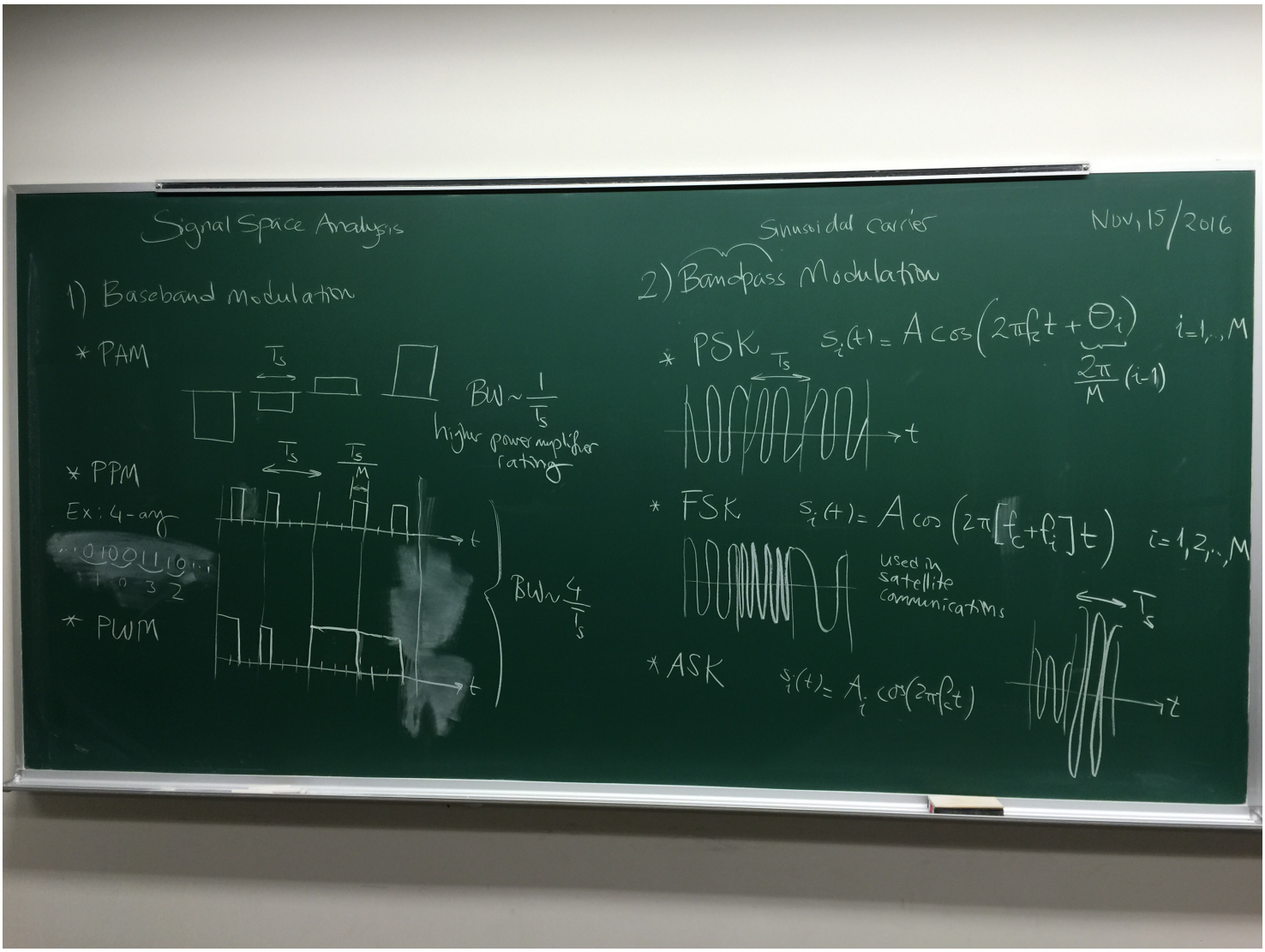
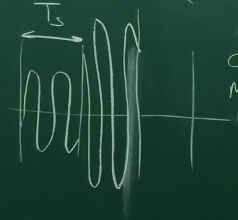

Signal Space Analysis
Nov, 15/2016
1) Baseband Modulation
* PAM
T_s
$BW \sim \frac{1}{T_s}$
higher power amplifier rating
* PPM
Ex: 4-ary
...0 1 0 0 1 1 1 0...
1 0 3 2
T_s
$\frac{T_s}{M}$
$BW \sim \frac{4}{T_s}$
* PWM
t
2) Bandpass Modulation
Sinusoidal carrier
* PSK
$s_i(t) = A\cos\left(2\pi f_c t + \underbrace{\theta_i}_{\frac{2\pi}{M}(i-1)}\right)$ $i=1,..,M$
T_s
* FSK
$s_i(t) = A\cos\left(2\pi[f_c + f_i]t\right)$ $i=1,2,..,M$
used in satellite communications
* ASK
$s_i(t) = A_i\cos(2\pi f_c t)$
T_s

The most common technique

ASK + PSK : QAM

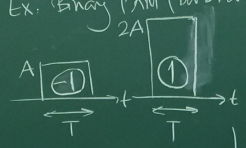
quadrature amplitude modulation

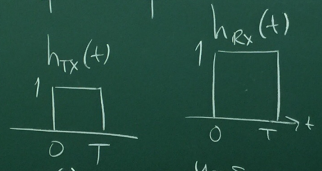
$s_i(t) = A_i \cos(2\pi f_c t + \theta_i)$

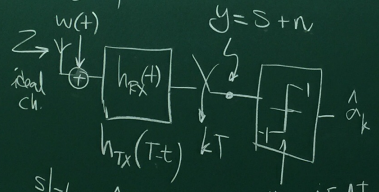


used in cellular, WiFi, microwave, wired modems

Ex. Binary PAM (baseband)







$s/-1 = AT$

$s/1 = 2AT$

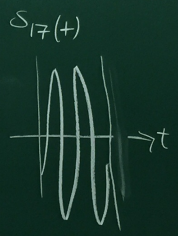
LTE 64QAM

$s_i(t) = A_i \cos(2\pi f_c t + \theta_i)$

* Receiver?
* Threshold levels @ the detector

} Difficult with the existing tools

→ Need signal space analysis



Quiz #3
Thu, Nov 17

2015, Quiz 3

R_{max}: $n\, W \log_2(1+SNR)$ — difficult to increase beyond 10

→ millimeter Wave comms
mmW are
f_c = 28 GHz, 60 GHz, 90 GHz } several GHz of BW available

Ex: Indoors

→ massive MIMO
@BS: $n \sim 1000$

$SE \triangleq \frac{R}{W}$

$SE_{max} = \frac{R_{max}}{W}$

$= n \log_2(1+SNR)$

SE per antenna pair

$SE_{max} = 6$ bits/sec/Hz

$10^7 = \min(2, n_{Rx}) \times W \times \log_2(1+100)$

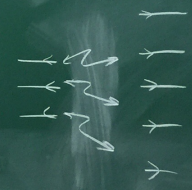
640 mm

$1.67\, 10^6 = \min(2, n_{Rx}) \times W$ — 6 bits/sym/Hz

$n_{Rx}=1 \rightarrow n=1 \rightarrow W = 1.67$ MHz

$n_{Rx}=2 \rightarrow n=2 \rightarrow W = 0.83$ MHz

$n_{Rx}>2 \rightarrow n=2 \rightarrow W = 0.83$ MHz



Signal Space Analysis

Waveforms ⟶ vectors

Ex: 

* length
* orientation (angle)

Dot Product

$$\bar{x}\cdot\bar{y} \triangleq \|\bar{x}_y\|\,\|\bar{y}\| = \|\bar{x}\|\,\|\bar{y}_x\|$$
$$= \|\bar{x}\|\,\|\bar{y}\|\cos\theta_{xy}$$
$$\theta_{xy} = \arccos\left(\frac{\bar{x}\cdot\bar{y}}{\|\bar{x}\|\,\|\bar{y}\|}\right)$$

$$\|\bar{x}\| = \sqrt{\bar{x}\cdot\bar{x}} \equiv \bar{x}\cdot\bar{x} = \|\bar{x}\|^2$$

$$(x(t), y(t)) \quad \langle x(t), y(t)\rangle \triangleq \int_{-\infty}^{\infty} x(t)y(t)\,dt$$

inner product

(measure of correlation, likeness)

$$(x(t), x(t)) = E_x$$

E_x: "length" squared

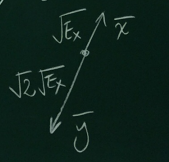
$$\theta_{xy} = \arccos\left(\frac{(x(t), y(t))}{\sqrt{E_x}\sqrt{E_y}}\right)$$

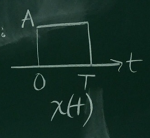
Ex: $E_x = A^2T \longrightarrow \|\bar{x}\| = A\sqrt{T}$

$E_y = 4A^2\frac{T}{2} = 2A^2T \quad \|\bar{y}\| = \sqrt{2}A\sqrt{T}$

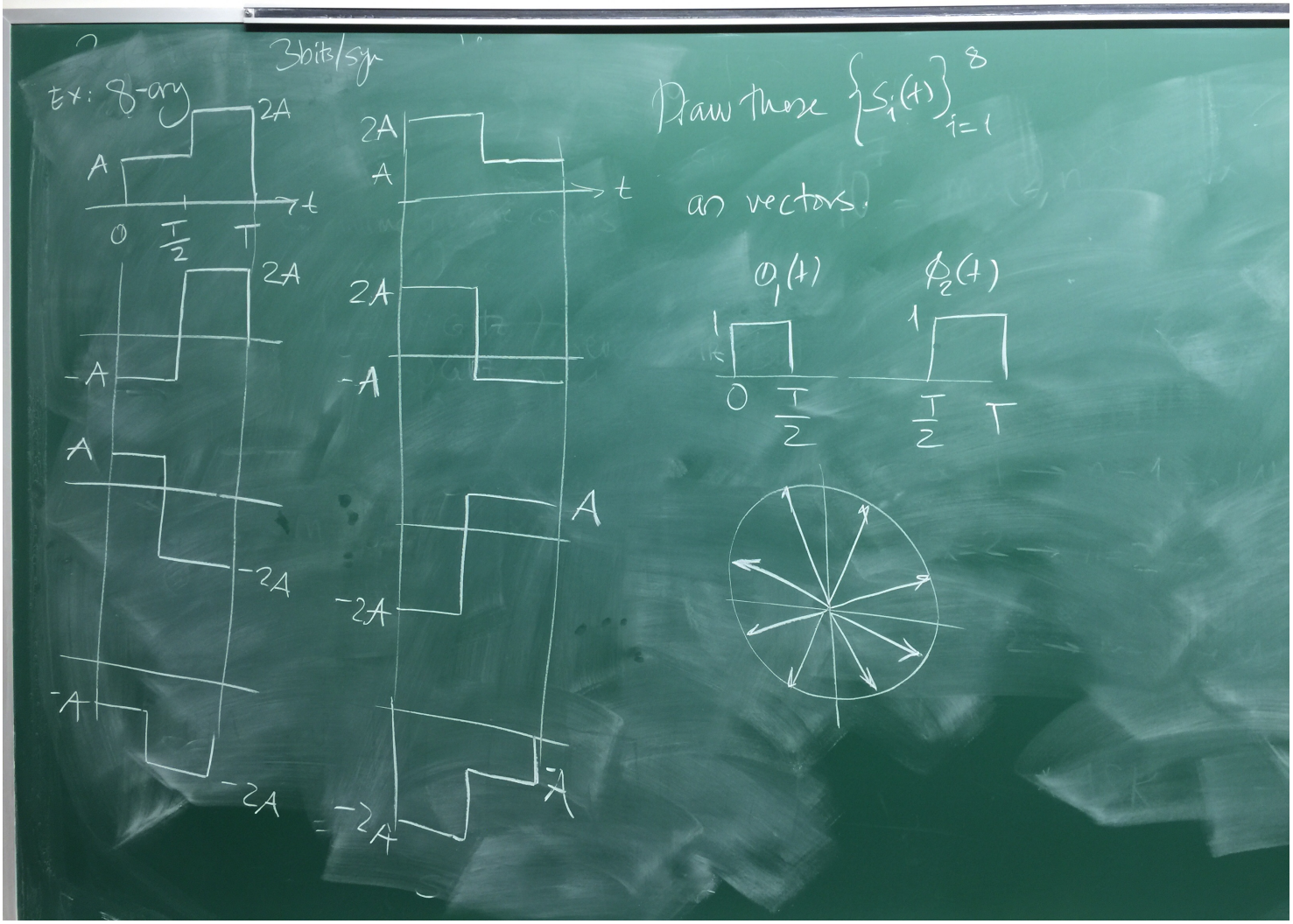
$$\theta_{xy} = \arccos\left(\frac{A^2T}{A\sqrt{T}\cdot\sqrt{2}A\sqrt{T}}\right) = 45^\circ$$

Find y(t) such that



Ex: 

2PAM


Ex: 8-ary
3bits/sym
2A
A
t
0
T/2
T
-A
-2A
Draw these $\{s_i(t)\}_{i=1}^{8}$
as vectors.
$\phi_1(t)$
$\phi_2(t)$
1