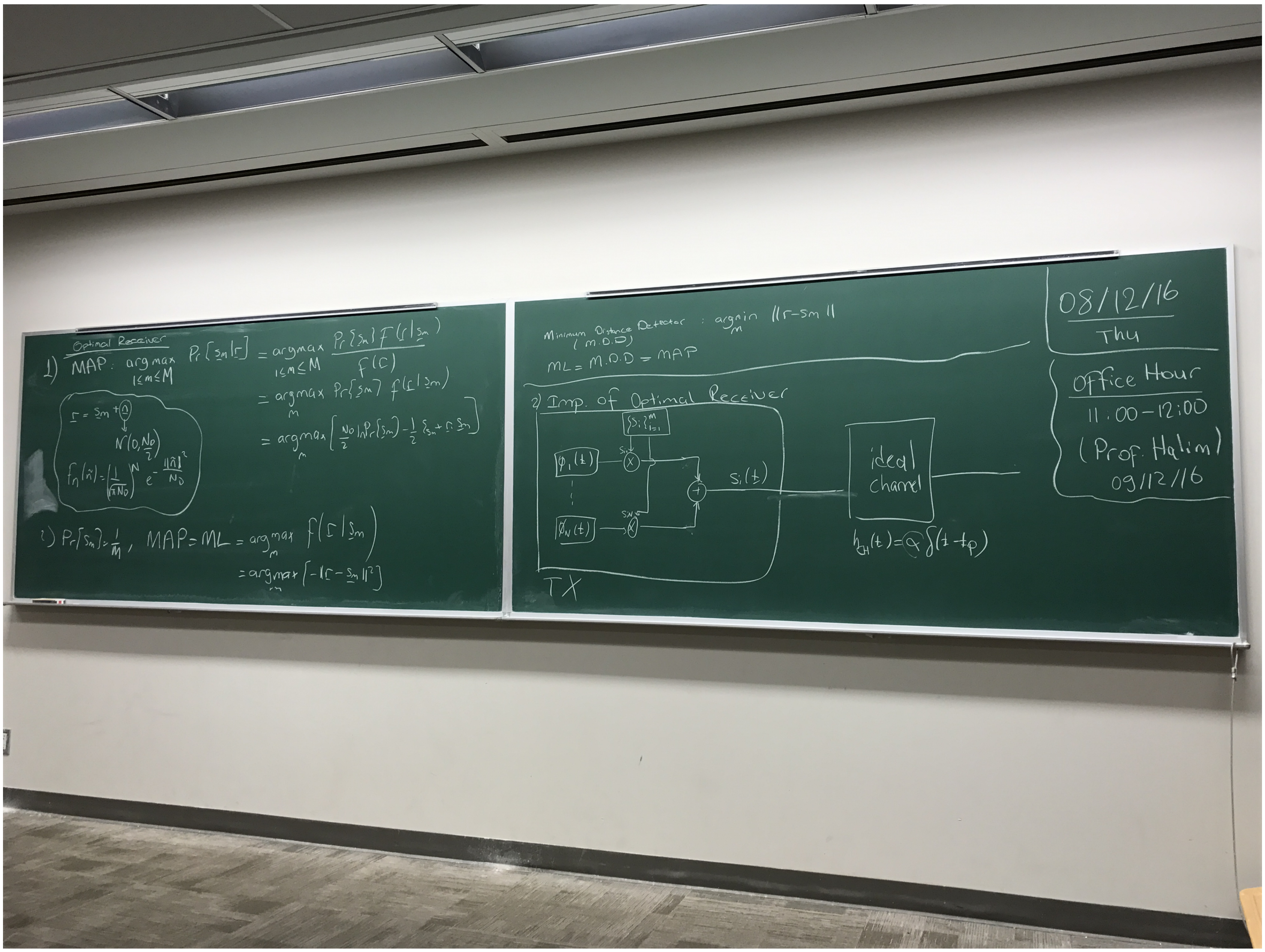
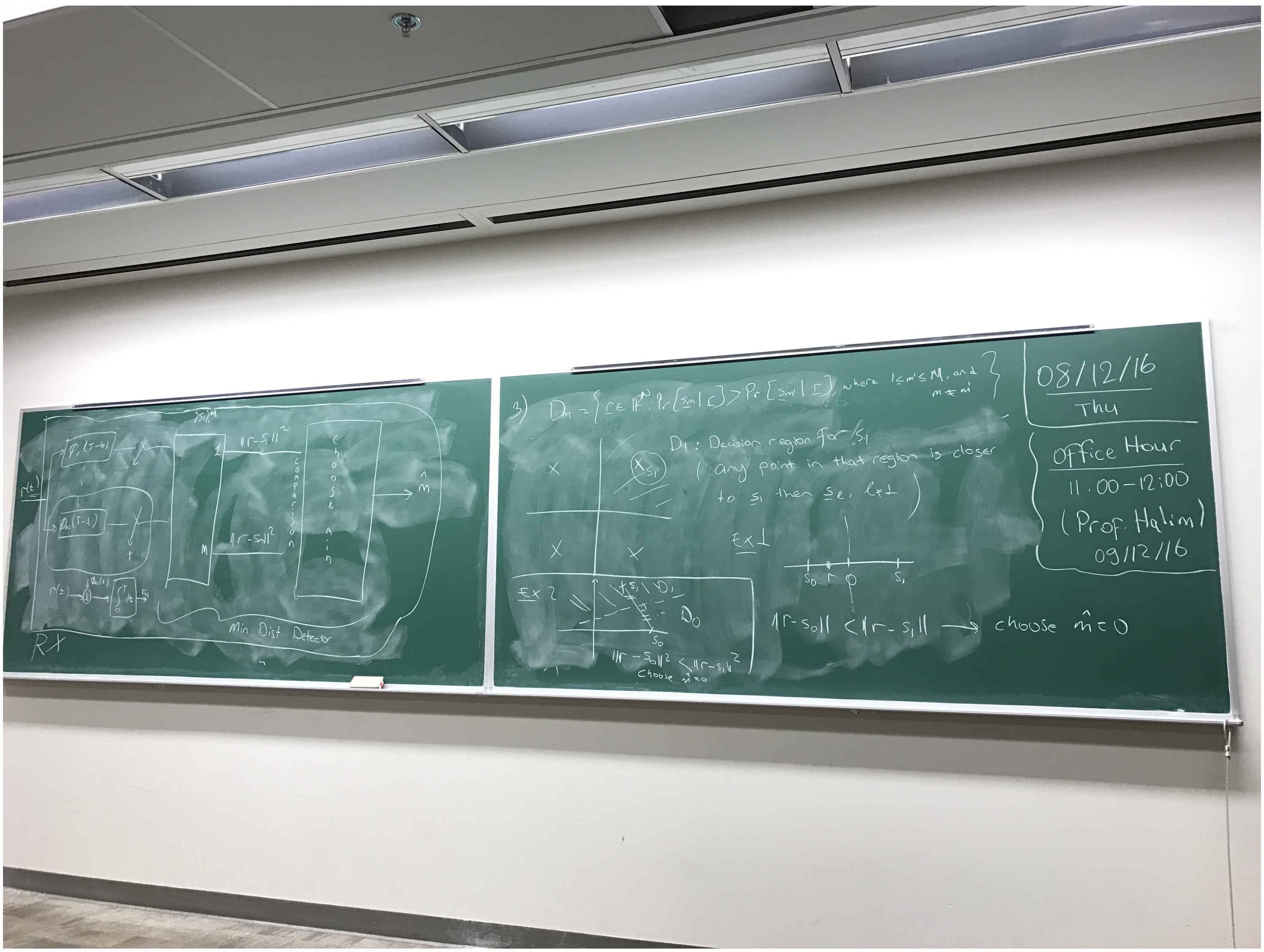
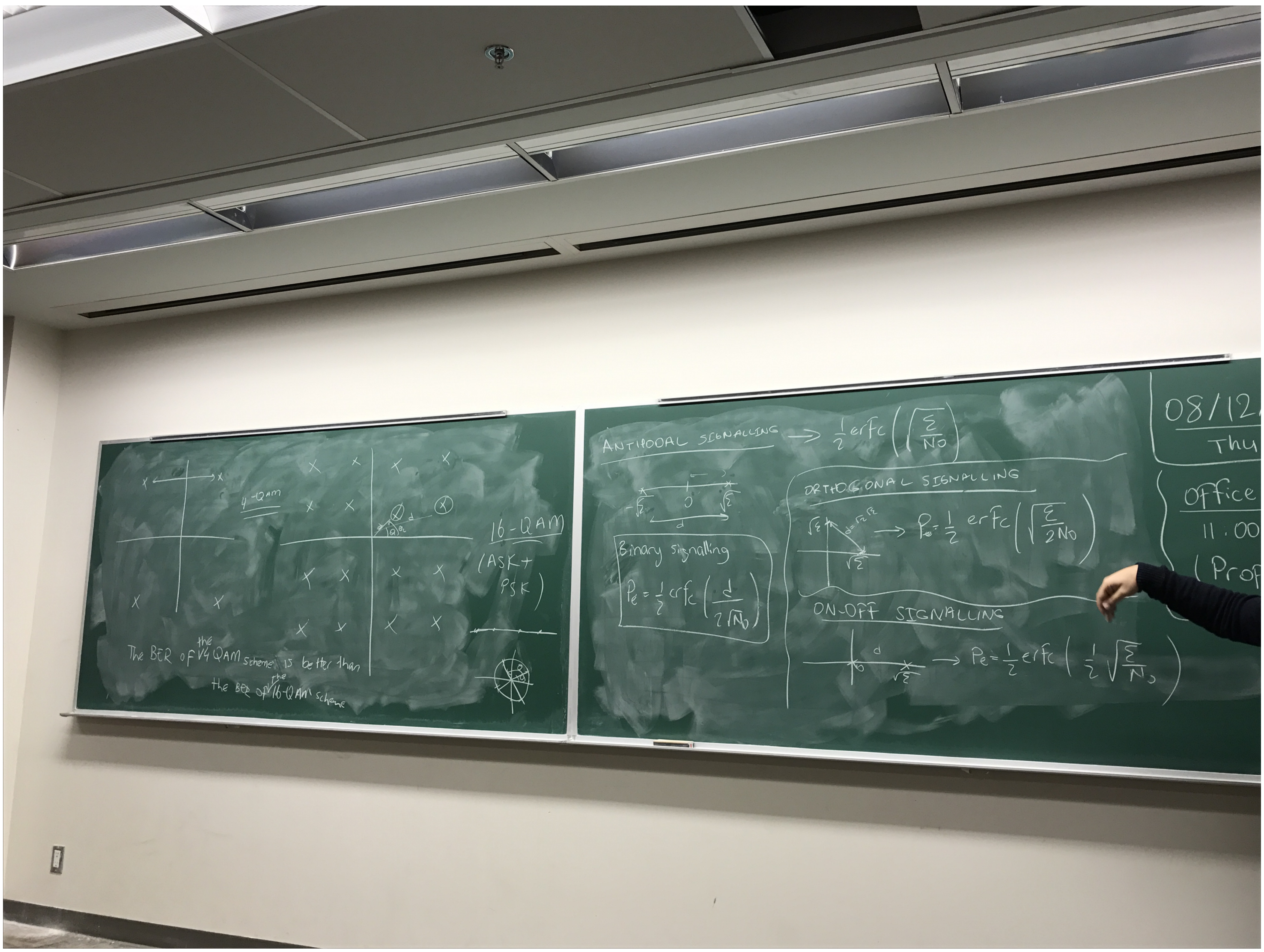


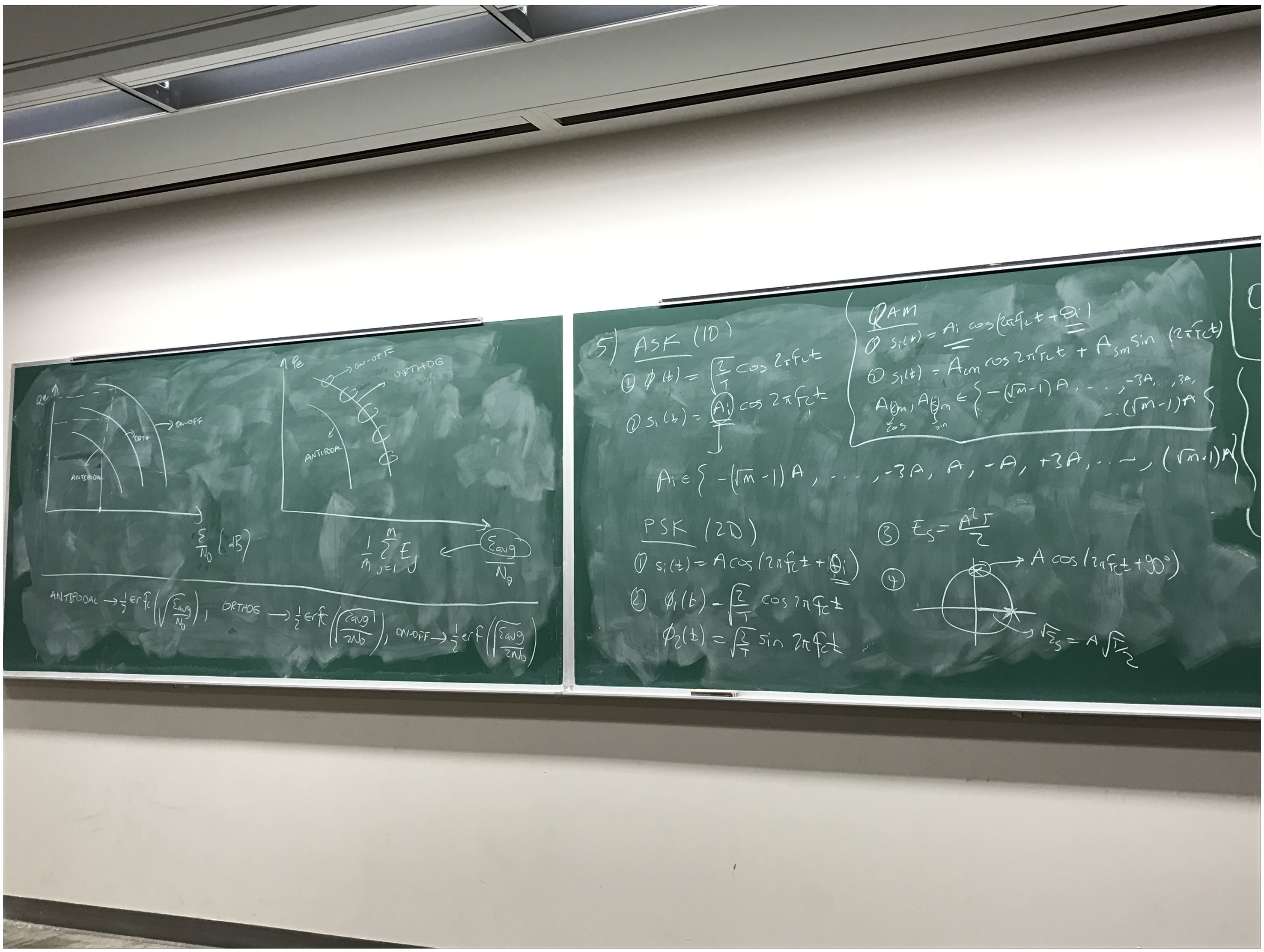
Optimal Receiver
1) MAP: $\arg\max_{1 \le m \le M} P_r\{\underline{s}_m \mid \underline{r}\} = \arg\max_{1 \le m \le M} \frac{P_r\{s_m\} f(\underline{r} \mid \underline{s}_m)}{f(\underline{r})}$
$= \arg\max_m P_r\{s_m\} f(\underline{r} \mid \underline{s}_m)$
$= \arg\max_m \left[ \frac{N_0}{2} \ln P_r\{\underline{s}_m\} - \frac{1}{2} \varepsilon_{s_m} + \underline{r} \cdot \underline{s}_m \right]$
$\underline{r} = \underline{s}_m + \underline{n}$
$N(0, \frac{N_0}{2})$
$f_n(\tilde{n}) = \left(\frac{1}{\sqrt{\pi N_0}}\right)^N e^{-\frac{\|\tilde{n}\|^2}{N_0}}$
2) $P_r\{s_m\} = \frac{1}{M}$, MAP = ML $= \arg\max_m f(\underline{r} \mid \underline{s}_m)$
$= \arg\max_m [-\|\underline{r} - \underline{s}_m\|^2]$
Minimum Distance Detector (M.D.D): $\arg\min_m \|r - s_m\|$
ML = M.D.D = MAP
2) Imp. of Optimal Receiver
$\{s_i\}_{i=1}^M$
$\phi_1(t)$
$s_{i1}$
$\phi_N(t)$
$s_{iN}$
$s_i(t)$
TX
ideal channel
$h_{CH}(t) = \alpha \delta(t - t_p)$
08/12/16
Thu
Office Hour
11:00–12:00
(Prof. Halim)
09/12/16



08/12/16
Thu
Office Hour
11.00-12:00
(Prof. Halim)
09/12/16
D1 : Decision region for s1
( any point in that region is closer
to s1 then
Ex 1
choose
Ex 2
D0
choose
comparison
choose min
Min Dist Detector
RX



4-QAM
16-QAM
(ASK+ PSK)
The BER of the 4 QAM scheme is better than the BER of the 16-QAM scheme
ANTIPODAL SIGNALLING
ORTHOGONAL SIGNALLING
Binary signalling
ON-OFF SIGNALLING
08/12
Thu
Office
11:00
(Prof



ON-OFF
ORTH
ANTIPODAL
$P_E$
ON-OFF
ORTHOG
ANTIPODAL
$\frac{\mathcal{E}}{N_0}$ (dB)
$\frac{1}{M}\sum_{j=1}^{M} E_j$
$\frac{\mathcal{E}_{avg}}{N_0}$
ANTIPODAL $\rightarrow \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_{avg}}{N_0}}\right)$, ORTHOG $\rightarrow \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_{avg}}{2N_0}}\right)$, ON-OFF $\rightarrow \frac{1}{2}\operatorname{erf}\left(\sqrt{\frac{\mathcal{E}_{avg}}{2N_0}}\right)$
5) ASK (1D)
① $\phi(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t$
② $s_i(t) = A_i \cos 2\pi f_c t$
$A_i \in \{-(\sqrt{M}-1)A, \ldots, -3A, A, -A, +3A, \ldots, (\sqrt{M}-1)A\}$
QAM
① $s_i(t) = A_i \cos(2\pi f_c t + \theta_i)$
② $s_i(t) = A_{cm} \cos 2\pi f_c t + A_{sm} \sin(2\pi f_c t)$
$A_{cm}, A_{sm} \in \{-(\sqrt{M}-1)A, \ldots, -3A, \ldots, 3A, \ldots (\sqrt{M}-1)A\}$
cos sin
PSK (2D)
① $s_i(t) = A\cos(2\pi f_c t + \theta_i)$
② $\phi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t$
$\phi_2(t) = \sqrt{\frac{2}{T}}\sin 2\pi f_c t$
③ $E_s = \frac{A^2 T}{2}$
④
$A\cos(2\pi f_c t + 90°)$
$\sqrt{E_s} = A\sqrt{T/2}$

(c) Prob of a symbol error

Prob of making correct decision

$$P_{c|m} = \int_{D_m} \underbrace{f_{\underline{r}}(\underline{r} \mid m \text{ sent})}_{\text{cond. prob}} \, d\underline{r}$$

$$f_{\underline{r}}(\underline{r} \mid m \text{ sent}) = \frac{1}{(\sqrt{\pi N_0})^N} \exp\left\{-\frac{1}{N_0}\sum_{k=0}^{N-1}(r_k - s_{m,k})^2\right\}$$

$$\underline{s}_m = \{s_{m,0}, s_{m,1}, \dots, s_{m,N-1}\}$$

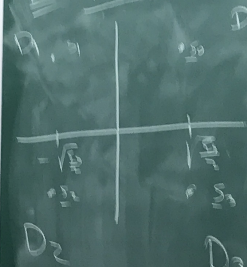
The average prob

$$P_c = \sum_{m=0}^{M-1} P_{c|m} \Pr\{m \text{ sent}\}$$

$$= \frac{1}{M}\sum_{m=0}^{M-1} P_{c|m}$$ (symbols with equal a priori prob)

$$\boxed{P_e = 1 - P_c}$$

Ex 4-QAM




$$s_0 = \left(\sqrt{\frac{E_s}{2}}, \sqrt{\frac{E_s}{2}}\right)$$

$$s_1 = \left(-\sqrt{\frac{E_s}{2}}, \sqrt{\frac{E_s}{2}}\right)$$

$$s_2 = \left(-\sqrt{\frac{E_s}{2}}, -\sqrt{\frac{E_s}{2}}\right)$$

$$s_3 = \left(\sqrt{\frac{E_s}{2}}, -\sqrt{\frac{E_s}{2}}\right)$$

$E_s$ = Energy per symbol

$$P_{c|0} = \int_{D_0} f_{\underline{r}}(\underline{r} \mid \text{"0" sent})\, d\underline{r}$$

$$= \int_0^{\infty}\int_0^{\infty} f_{r_0,r_1}(r_0, r_1 \mid \text{"0" sent})\, dr_0\, dr_1$$

$$= \int_0^{\infty}\int_0^{\infty} \frac{1}{\pi N_0} \exp\left\{-\frac{1}{N_0}\left[\left(r_0 - \sqrt{\frac{\mathcal{E}_s}{2}}\right)^2 + \left(r_1 - \sqrt{\frac{\mathcal{E}_s}{2}}\right)^2\right]\right\} \cdot dr_0 \cdot dr_1$$

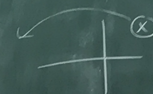
$$= \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{N_0}\left[\left(r_0 - \sqrt{\frac{\mathcal{E}_s}{2}}\right)^2\right]\right\} \cdot dr_0 \cdot \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{N_0}\left[\left(r_1 - \sqrt{\frac{\mathcal{E}_s}{2}}\right)^2\right]\right\} \cdot dr_1$$

$$= \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\sqrt{\frac{\mathcal{E}_s}{2N_0}}} e^{-u_0^2}\, du_0\right]\left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\sqrt{\frac{\mathcal{E}_s}{2N_0}}} e^{-u_1^2}\, du_1\right]$$

where $u_0 = -\frac{r_0 - \sqrt{\frac{\mathcal{E}_s}{2}}}{\sqrt{N_0}}$

$u_1 = -\frac{r_1 - \sqrt{\frac{\mathcal{E}_s}{2}}}{\sqrt{N_0}}$

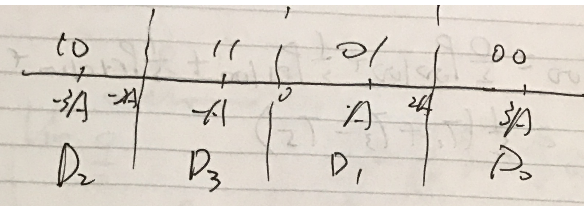
$$= \left(1 - \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{\mathcal{E}_s}{2N_0}}}^{\infty} e^{-u^2}\, du\right)^2$$



$$P_{c|0} = \left(1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_s}{2N_0}}\right)\right)^2$$

$$\boxed{P_{c|m} = \left[1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_s}{2N_0}}\right)\right]^2} \rightarrow \text{due to symmetric nature of}$$

$$P_e = 1 - \frac{1}{4}\sum_{m=0}^{3} P_{c|m} = 1 - \left[1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_s}{2N_0}}\right)\right]^2 \cong \operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_s}{2N_0}}\right)$$




If 00 was transmitted then $r = 3A + w$.

$$f_r(r/00 \text{ sent}) = \frac{1}{\sqrt{\pi N_0}} \exp\left\{ -\frac{1}{N_0}(r-3A)^2 \right\}$$

$$\begin{aligned}
P_{00/00} &= P_r(r > 2A / 00 \text{ sent}) \\
&= P_r\{W > -A\} \\
&= 1 - P_r\{W < -A\} \\
&= 1 - P_r\{W > A\} = 1 - \underbrace{\tfrac{1}{2}\operatorname{erfc}\left(\tfrac{A}{\sqrt{N_0}}\right)}_{T_1} \\
&= 1 - T_1
\end{aligned}$$

$$T_n = \frac{1}{2}\operatorname{erfc}\left(\frac{nA}{\sqrt{N_0}}\right) = P_r\{W < -nA\} = P_r\{W > nA\}$$

$$P_{01/00} = P_r\{0 < r < 2A / 00 \text{ sent}\} = P_r\{-3A \le W \le -A\} = T_1 - T_3$$

$$P_{11/00} = P_r\{-2A < r < 0 / 00 \text{ sent}\} = P_r\{-5A < W < -3A\} = T_3 - T_5$$

$$P_{10/00} = P_r\{r < -2A / 00 \text{ sent}\} = P_r\{W \le -5A\} = T_5$$

$$P_{b/00} = \sum_{\ell=0}^{M-1} P_r\{\hat{k} = \ell / 00 \text{ sent}\} \times \left\{ \frac{\text{\# of bit positions in which 00 and } \ell \text{ differ}}{\text{\# of bits per symbol}} \right\}$$

$$P_{b|00} = \frac{0}{2} P_{00|00} + \frac{1}{2} P_{01|00} + \frac{1}{2} P_{10|00} + \frac{2}{2} P_{11|00}$$

$$= \frac{1}{2}(T_1 + T_3 - T_5)$$

$- T_3) + T_5 + T_3 - T_5$

$\frac{1}{2} T_1 - \frac{1}{2} T_3 + T_3 -$