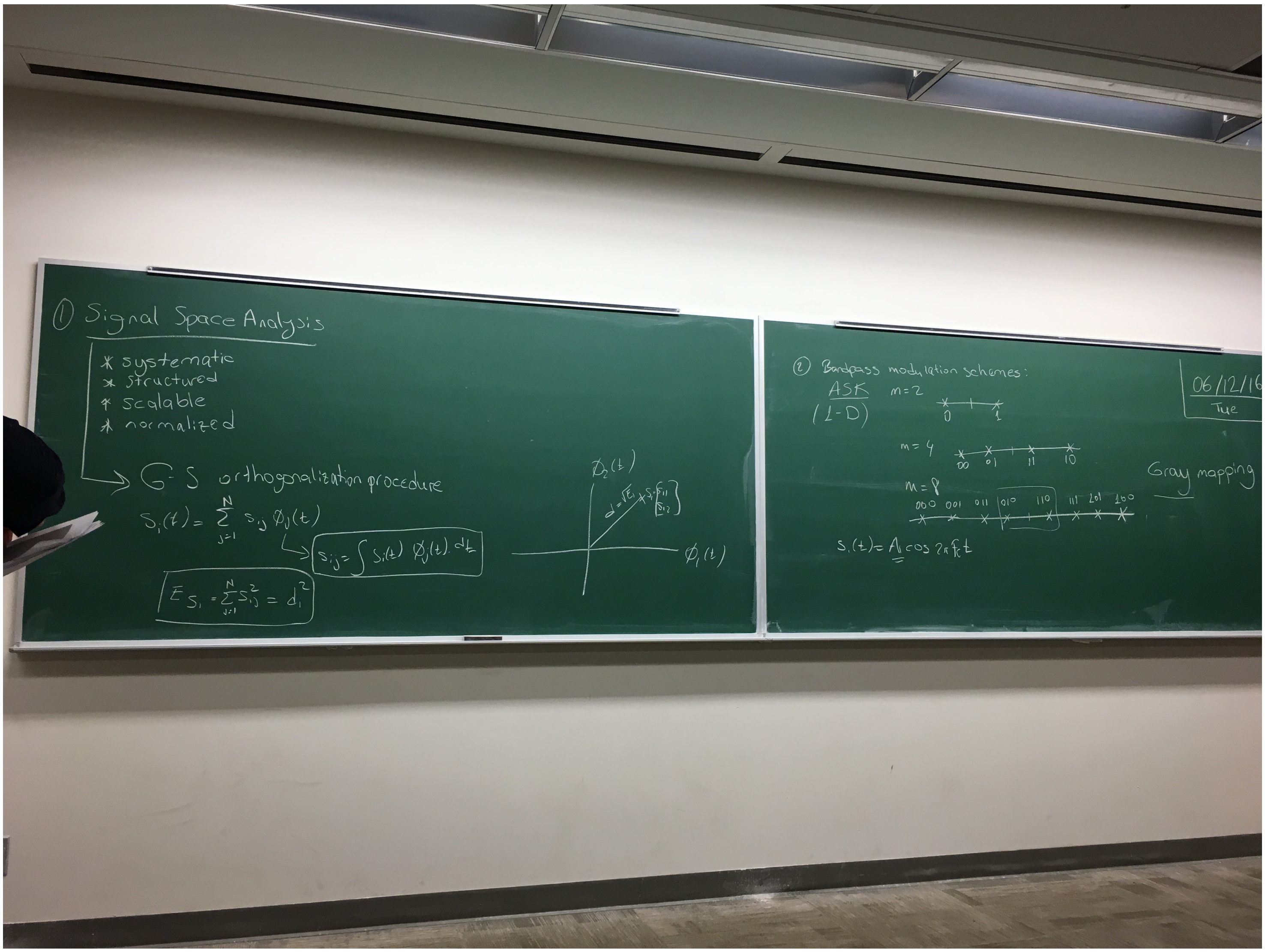
① Signal Space Analysis
* systematic
* structured
* scalable
* normalized
G-S orthogonalization procedure
$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t)$
$\boxed{s_{ij} = \int s_i(t)\,\phi_j(t)\,dt}$
$\boxed{E_{s_i} = \sum_{j=1}^{N} s_{ij}^2 = d_i^2}$
$\phi_2(t)$
$d = \sqrt{E_i}$
$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix}$
$\phi_1(t)$
② Bandpass modulation schemes:
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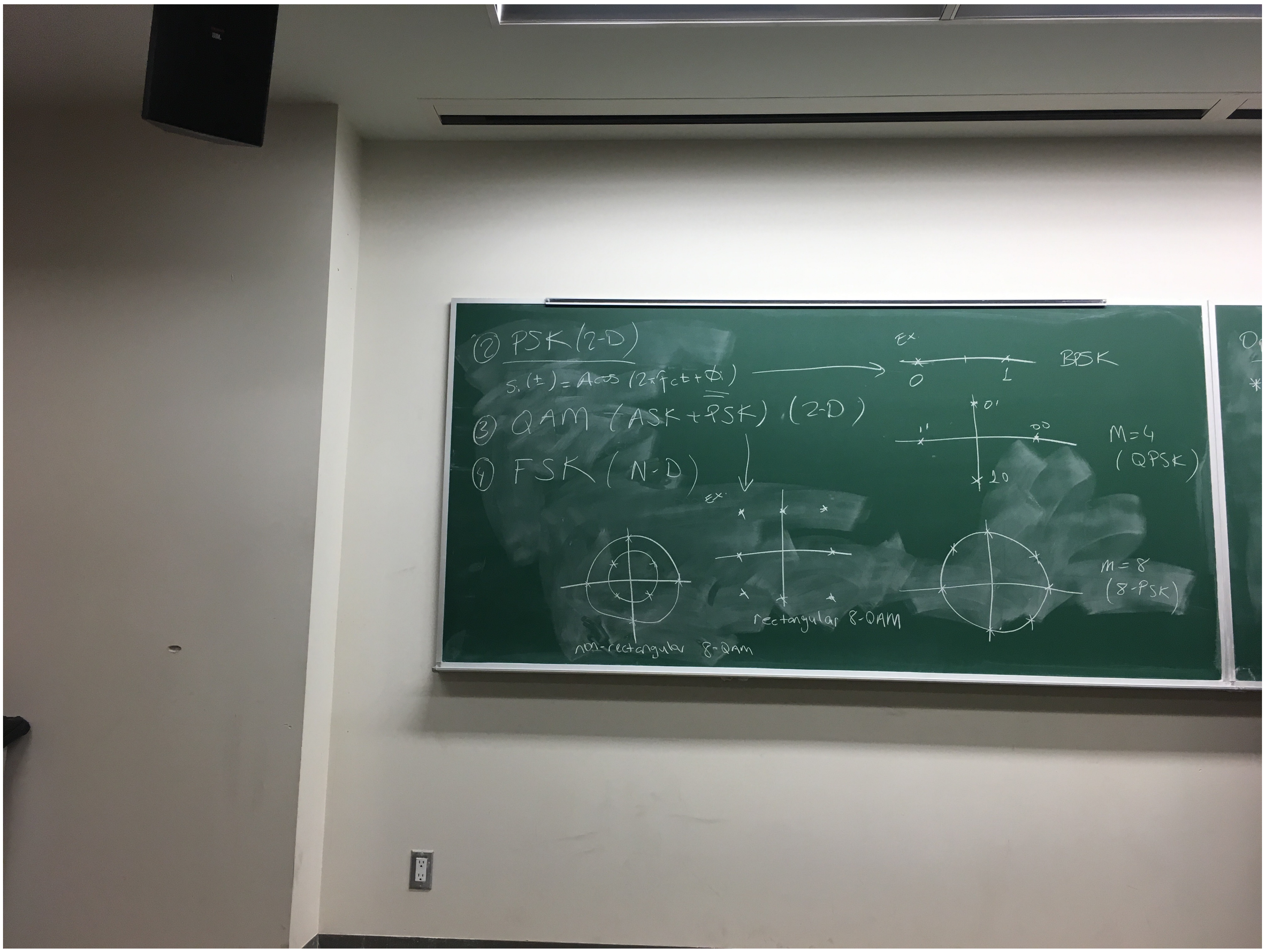
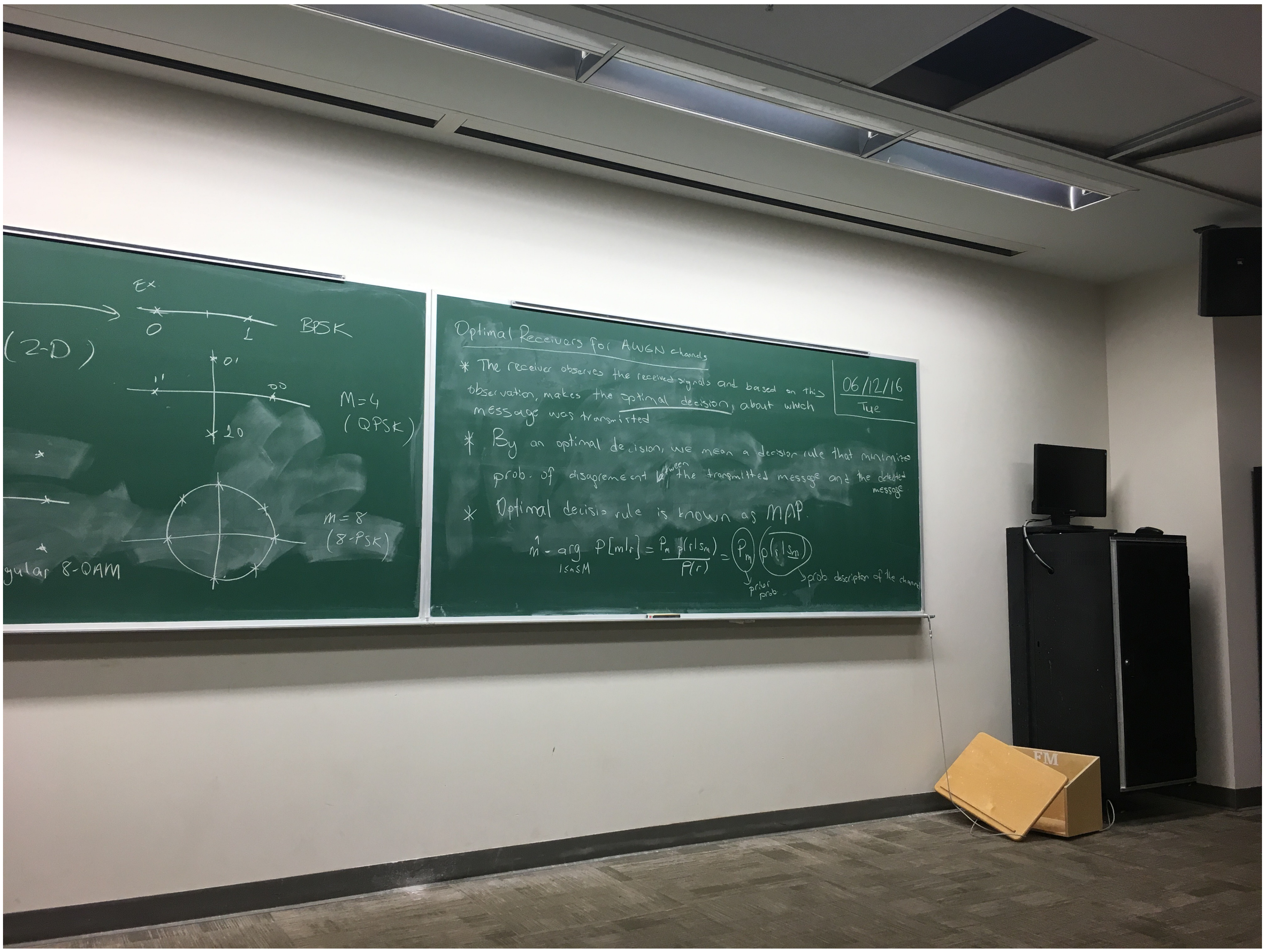
ASK
(1-D)
m=2
0 1
m=4
00 01 11 10
m=8
000 001 011 010 110 111 101 100
Gray mapping
$s_i(t) = A_i \cos 2\pi f_c t$


② PSK (2-D)
s_i(t) = A cos (2πf_c t + Φ_i)
Ex
BPSK
0
1
③ QAM (ASK + PSK) (2-D)
01
11
00
10
M=4
(QPSK)
④ FSK (N-D)
Ex
rectangular 8-QAM
non-rectangular 8-QAM
M=8
(8-PSK)


Ex
BPSK
(2-D)
M=4
(QPSK)
m=8
(8-PSK)
Optimal Receivers for AWGN channels
* The receiver observes the received signals and based on this observation, makes the optimal decision, about which message was transmitted
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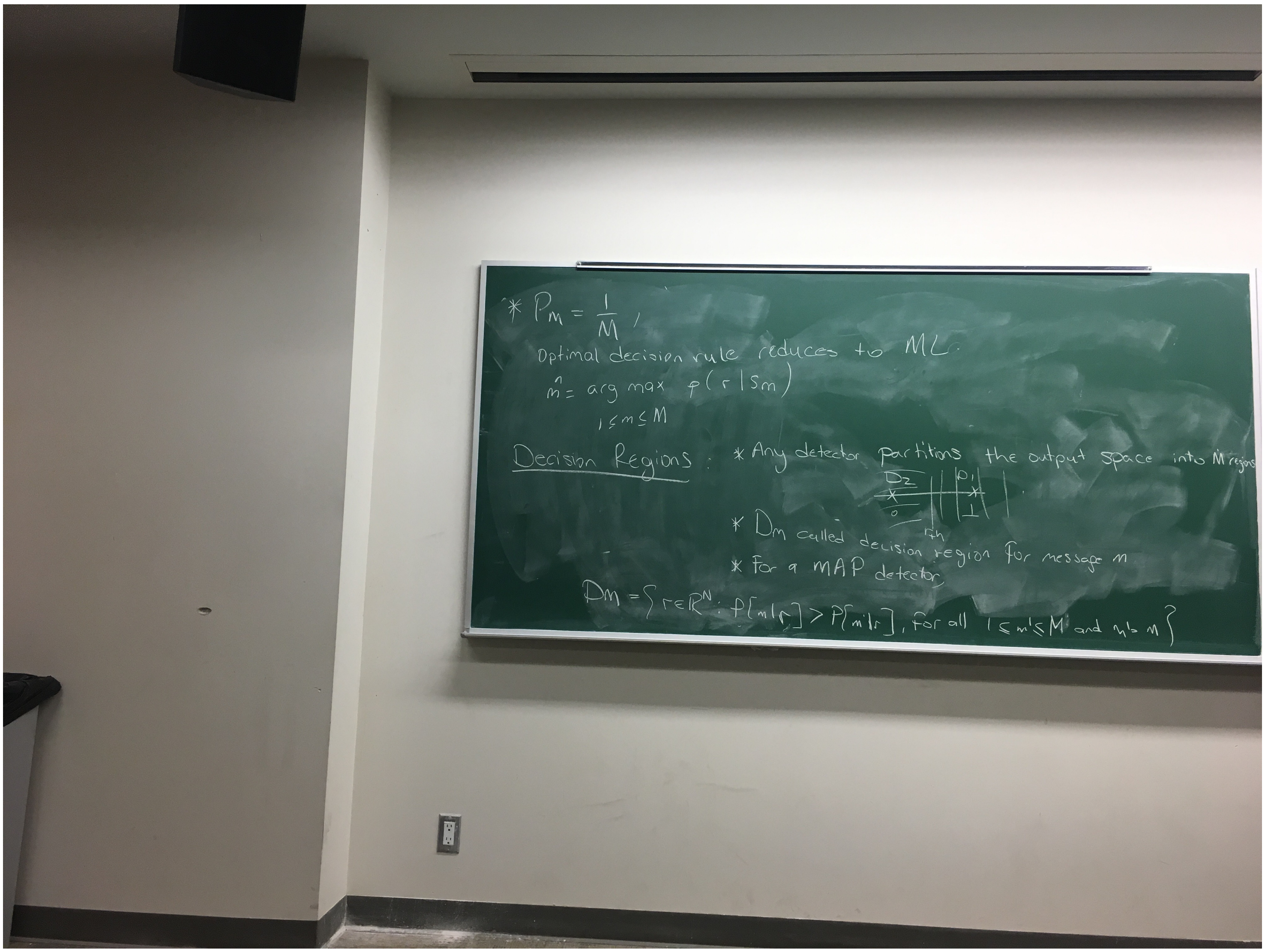
* By an optimal decision, we mean a decision rule that minimizes prob. of disagreement between the transmitted message and the detected message.
* Optimal decision rule is known as MAP.
$\hat{m} = \arg_{1 \le m \le M} P[m|r] = \frac{P_m \, p(r|s_m)}{P(r)} = P_m \, p(r|s_m)$
prior prob
prob. description of the channel


* $P_m = \frac{1}{M}$,
Optimal decision rule reduces to ML.
$\hat{m} = \arg\max_{1 \le m \le M} p(r \mid s_m)$
Decision Regions:
* Any detector partitions the output space into M regions
D_2
D_1
* D_m called decision region for message m^{th}
* For a MAP detector,
$D_m = \{ r \in \mathbb{R}^N : P[m \mid r] > P[m' \mid r], \text{ for all } 1 \le m' \le M \text{ and } m' \ne m \}$


MAP Detector
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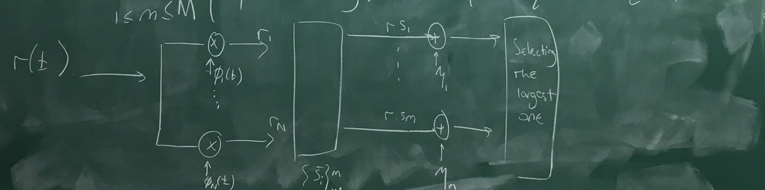
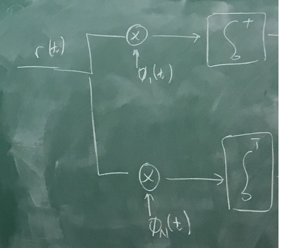
the output space into M regions
region for message m
D_1
D_2
D_3
D_4
s_1
s_2
s_3
s_4
minimum-distance detector

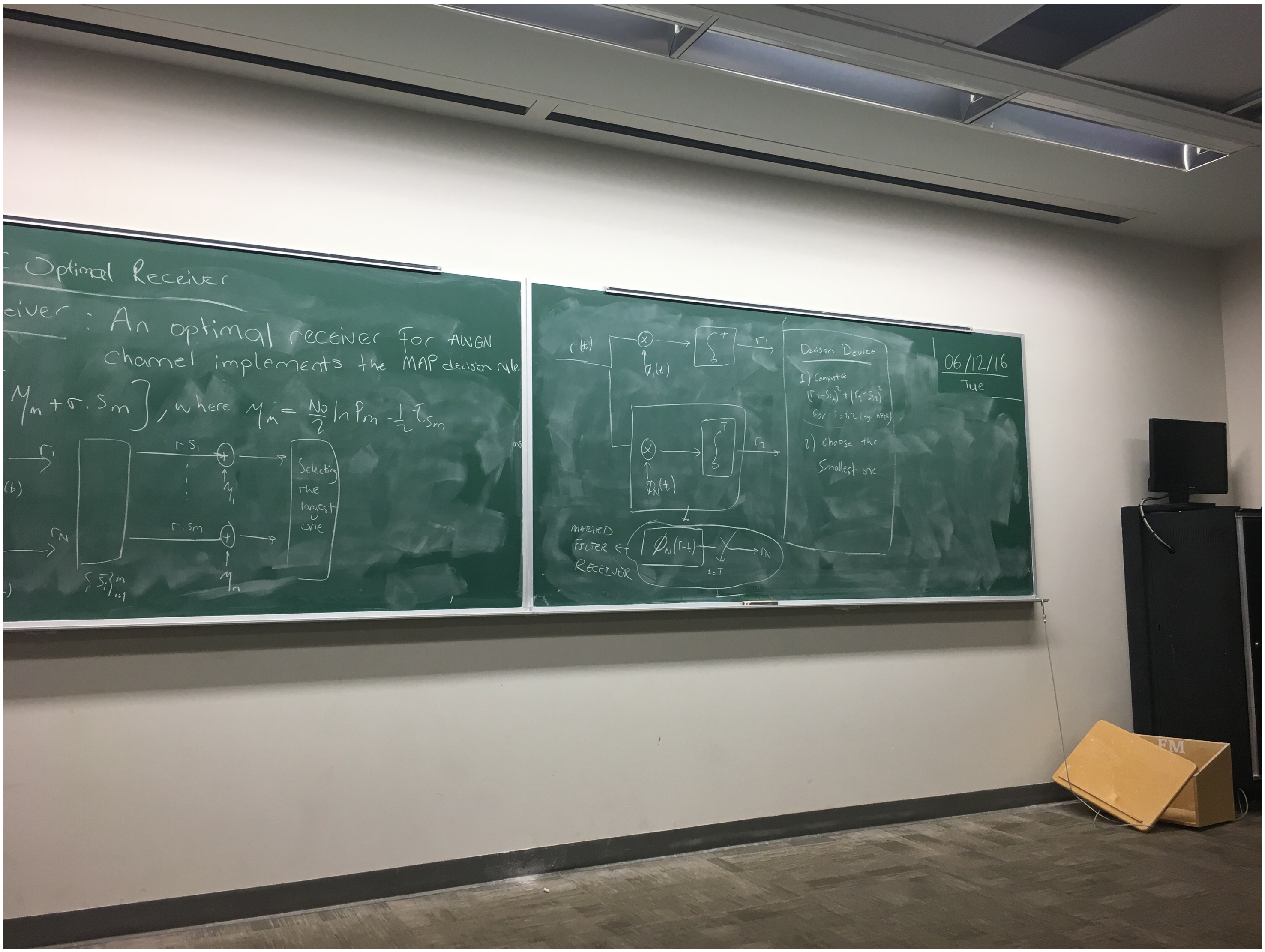
Implementation of Optimal Receiver

Correlation Receiver: An optimal receiver for AWGN channel implements the MAP decision rule

$$\hat{m} = \underset{1 \le m \le M}{\arg\max} \left[\eta_m + r \cdot s_m \right], \text{ where } \eta_m = \frac{N_0}{2} \ln P_m - \frac{1}{2} \mathcal{E}_{s_m}$$






Optimal Receiver
An optimal receiver for AWGN channel implements the MAP decision rule
where
Selecting the largest one
Decision Device
1) Compute
2) Choose the Smallest one
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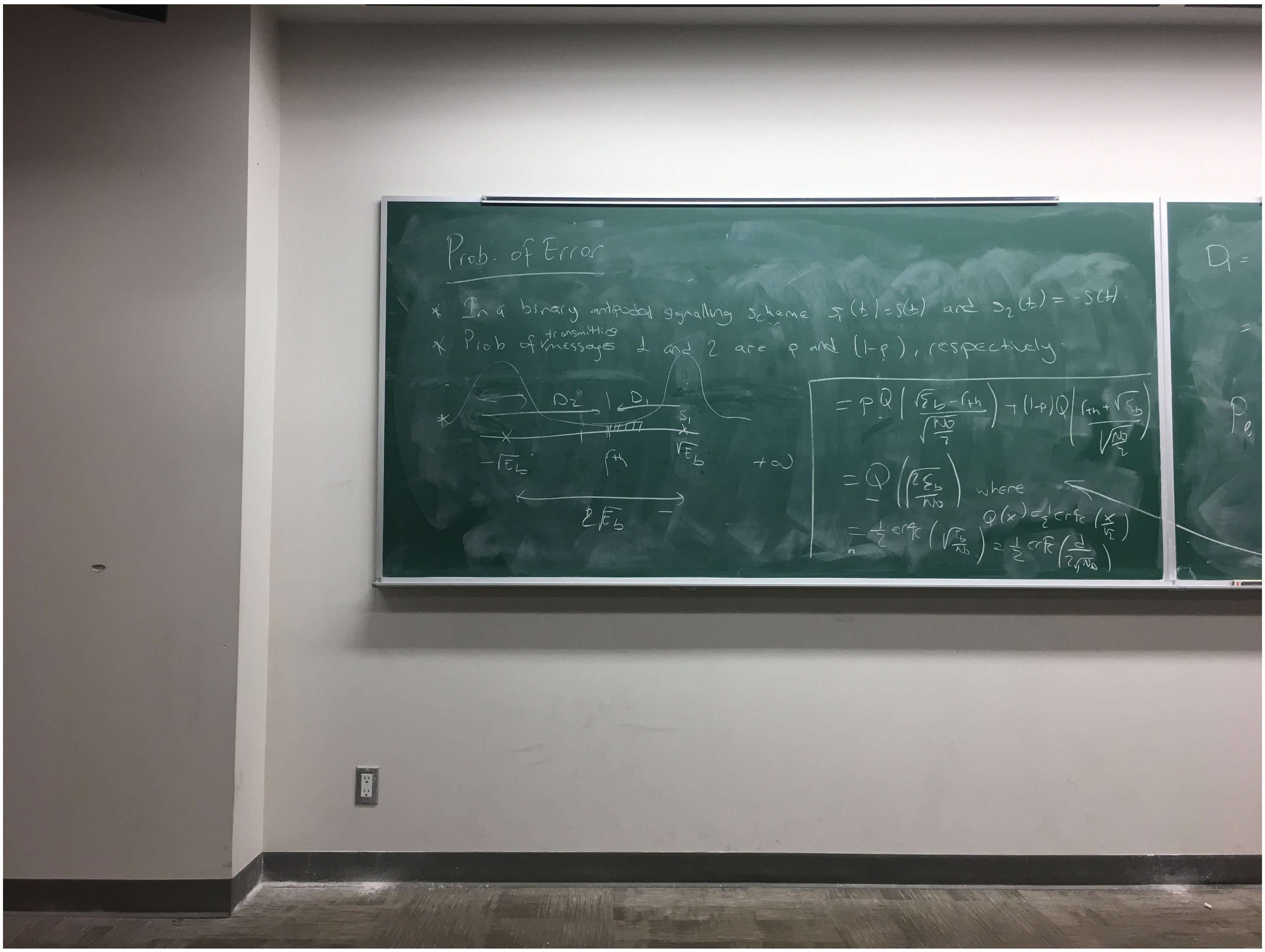
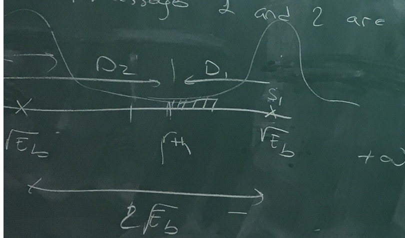
MATCHED FILTER RECEIVER



Prob. of Error
* In a binary antipodal signalling scheme $s_1(t) = s(t)$ and $s_2(t) = -s(t)$
* Prob of transmitting messages 1 and 2 are p and $(1-p)$, respectively
D_2
D_1
s_1
$-\sqrt{E_b}$
r_{th}
$\sqrt{E_b}$
$+\infty$
$2\sqrt{E_b}$
$= p\,Q\left(\frac{\sqrt{\varepsilon_b}-r_{th}}{\sqrt{\frac{N_0}{2}}}\right) + (1-p)\,Q\left(\frac{r_{th}+\sqrt{\varepsilon_b}}{\sqrt{\frac{N_0}{2}}}\right)$
$= Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)$ where
$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$
$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\varepsilon_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d}{2\sqrt{N_0}}\right)$
$D_1 =$
P_e

b. of Error

n a binary antipodal signalling scheme $s_1(t) = s(t)$ and $s_2(t) = -s(t)$.
ob of transmitting messages 1 and 2 are p and $(1-p)$, respectively.



$+\infty$

$$\boxed{\begin{aligned} &= p\,Q\left(\frac{\sqrt{\mathcal{E}_b} - r_{th}}{\sqrt{\frac{N_0}{2}}}\right) + (1-p)\,Q\left(\frac{r_{th} + \sqrt{\mathcal{E}_b}}{\sqrt{\frac{N_0}{2}}}\right) \\ &= Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) \text{ where } Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{d}{2\sqrt{N_0}}\right) \end{aligned}}$$

$$D_1 = \left\{ r : r\sqrt{\mathcal{E}_b} + \frac{N_0}{2}\ln p - \frac{1}{2}\mathcal{E}_b > -r\sqrt{\mathcal{E}_b} + \frac{N_0}{2}\ln(1-p) - \frac{1}{2}\mathcal{E}_b \right\}$$

$$= \left\{ r : r > \underbrace{\frac{N_0}{4\sqrt{\mathcal{E}_b}} \ln\frac{(1-p)}{p}}_{r_{th}} \right\}$$

06/12/16
Tue

$$P_e = p \int_{D_2} f(r \mid s_1 = \sqrt{\mathcal{E}_b}) \cdot dr + (1-p) \int_{D_1} f(r \mid s_2 = -\sqrt{\mathcal{E}_b}) \cdot dr$$

$$D_2 = \int_{-\infty}^{r_{th}}, \quad D_1 = \int_{r_{th}}^{\infty}$$

$$= p\,P_r\left[N\left(\sqrt{\mathcal{E}_b}, \frac{N_0}{2}\right) < r_{th}\right] + (1-p)\,P_r\left[N\left(-\sqrt{\mathcal{E}_b}, \frac{N_0}{2}\right) > r_{th}\right]$$