Signal Space Analysis

- Systematic
- Structured
- Scalable
- Normalized

GS: orthogonalization procedure

$$S_{ij} = \sum_{k=1}^{N} y_k(t)$$

$$z_{ij} = \sum_{k=1}^{N} x_k(t) y_k(t)$$

$$E_{S_i} = \sum_{j=1}^{N} |z_{ij}|^2 = d_j$$

$$2\pi$$
1. PSK (2-D)
   \[ s(t) = A \cos (2\pi f_t + \phi) \]

2. QAM (ASK + FSK) (2D)

3. FSK (N-D)

Diagram: Rectangular 2-DAM

M = 4 (QPSK)
M = 8 (8-PSK)
Optimal Recovery for Amplification

- The receiver recovers the transmitted data based on the observation, makes the optimal decision, and outputs which message was transmitted.
- By an optimal decision rule, one determines the maximum probability of error of the transmitted message.
- Optimal decision rule is known as MPP.

\[ P[\text{error}] = P[\text{wrong message}] = \frac{P[\text{message not recovered}]}{P[\text{message recovered}]} \]

Diagram:
- QPSK
- 8-QAM
- 16-QAM
\[ P_m = \frac{1}{M} \]

Optimal decision rule reduces to ML.

\[ m = \arg \max_{m \in M} P(Y | m) \]

Decision Regions:

- Any detector partitions the output space into regions of the form \( D_m \). For a MAP detector:

\[ D_m = \{ y \in \mathbb{R}^n \mid P(y | m) > P(y | m') \text{ for all } m' \neq m \text{ and } y \} \]
Map the output space into a region for message m.
Implementation of Optimal Receiver

Correlation Receiver: An optimal receiver for an AWGN channel implements the MAP decision rule.

\[ m = \arg \max_{m \in \mathbb{C}^M} \left( M_{m} + R_{m} \right) \]

where \( m = \frac{N_{0} / 2}{\sigma_{m}^2} - I_{m} \)

[Diagram of correlation receiver]
**Optimal Receiver**

An optimal receiver for an AWGN channel implements the MAP decision rule:

\[ M_n + r.s.m \]

where \( M_n = \frac{1}{2} \cdot h \cdot p_m - I_m \).
Prob. of Error

1. In a binary antipodal signaling scheme, \( P(1|x_i) = x_i \) and \( P(0|x_i) = 1-x_i \).
2. Prob of messages 1 and 2 are \( p \) and \( 1-p \), respectively.

\[
\begin{align*}
E_b &= \frac{1}{2} \left[ \frac{P(1|x_i)}{1-P(1|x_i)} + \frac{P(0|x_i)}{1-P(0|x_i)} \right] \\
 &= \frac{1}{2} \left[ \frac{x_i}{1-x_i} + \frac{1-x_i}{1+x_i} \right] \\
 &= \frac{1}{2} \left[ \frac{x_i + (1-x_i)}{1-x_i} \right] \\
&= \frac{1}{2} \left[ \frac{1}{1-x_i} \right] \\
&= \frac{1}{2} \frac{1}{1-x_i}
\end{align*}
\]
b. of Error

In a binary extended signaling scheme $S_1(10,11)$ and $S_2(10,11)$, the probabilities of messages 1 and 2 are $\phi_1$ and $\phi_2$, respectively.

\[
D_2 = \text{Pr}\left[ \sqrt{2} \left( \left| E_1 - E_2 \right| \right) \right]
\]
\[
= \phi_1 \left( \frac{E_1 + E_2}{\sqrt{2}} \right) + \phi_2 \left( \frac{E_1 - E_2}{\sqrt{2}} \right)
\]
\[
= \Phi \left( \frac{E_1 + E_2}{\sqrt{2}} \right) + \Phi \left( \frac{E_1 - E_2}{\sqrt{2}} \right)
\]
where
\[
E_1 = E_2
\]
\[
= \frac{1}{\sqrt{2}} \left( E_1 + E_2 \right)
\]
\[
= \frac{1}{\sqrt{2}} \left( E_1^2 + E_2^2 \right)
\]
\[
= \frac{1}{\sqrt{2}} \left( E_1^2 + E_2^2 \right)
\]