Labeling = Bit-to-symbol mapping

* Natural labeling
  (use the symbol index)

\[ S_{2^3} = 110 \]

\[ P_b = \frac{1}{2^3} P_s \]

Ex. \[ P_s = 10^{-4} \]

10^6 symbols = 3\times10^6 bits

\[ P_b = \frac{100}{3\times10^6} = \frac{1}{3}\times10^{-7} \]

if 000 is transmitted the most likely erroneous decision are 111 and 001

Set \( m = m_i \) if \( ||r - s_i|| \) is minimum and \( i = 1 \)

Fitting out:
projecting node on the signal space;
rejecting noise on all other dimensions.
Optimum Decision Rule

Set \( \hat{m} = m_i \) if
\[
P(m_i \text{ is } x_i \mid \mathbf{r}) \text{ is maximized for } j = i
\]

Maximum A Posteriori (MAP) rule

Bayes' Rule
\[
P(A \mid B) = \frac{P(B \mid A) P(A)}{P(B)} = \frac{P(A \mid B) P(B)}{P(A) P(B)}
\]

\[
P(m_j \mid x_i) = \frac{P(x_i \mid m_j) P_{m_j}}{P(x_i)} \propto \frac{f_i (m_j) P_{m_j}}{P(i)}
\]

constant
Set \( \hat{m} = m \_j \) if
\[
f(r | m_\_j) \text{ is maximized for } j = \_i
\]
Maximum Likelihood principle

\[
\text{MAP} = \text{ML}
\]
if all symbols are equally-likely

Set \( \hat{m} = m \) if
\[
\| r - \hat{m} \| \text{ is minimized for } j = \_i
\]
Minimum distance rule

Log likelihood
\[
-\frac{1}{N} \sum_{i=1}^{N} (r_i - \hat{m}_i)^2
\]

\[
= \text{minimizing } \sum (r_i - \hat{m}_i)^2
\]

When \( r \) is in region \( C_1 \)
\[
\hat{m} = m
\]

Netflix example learning