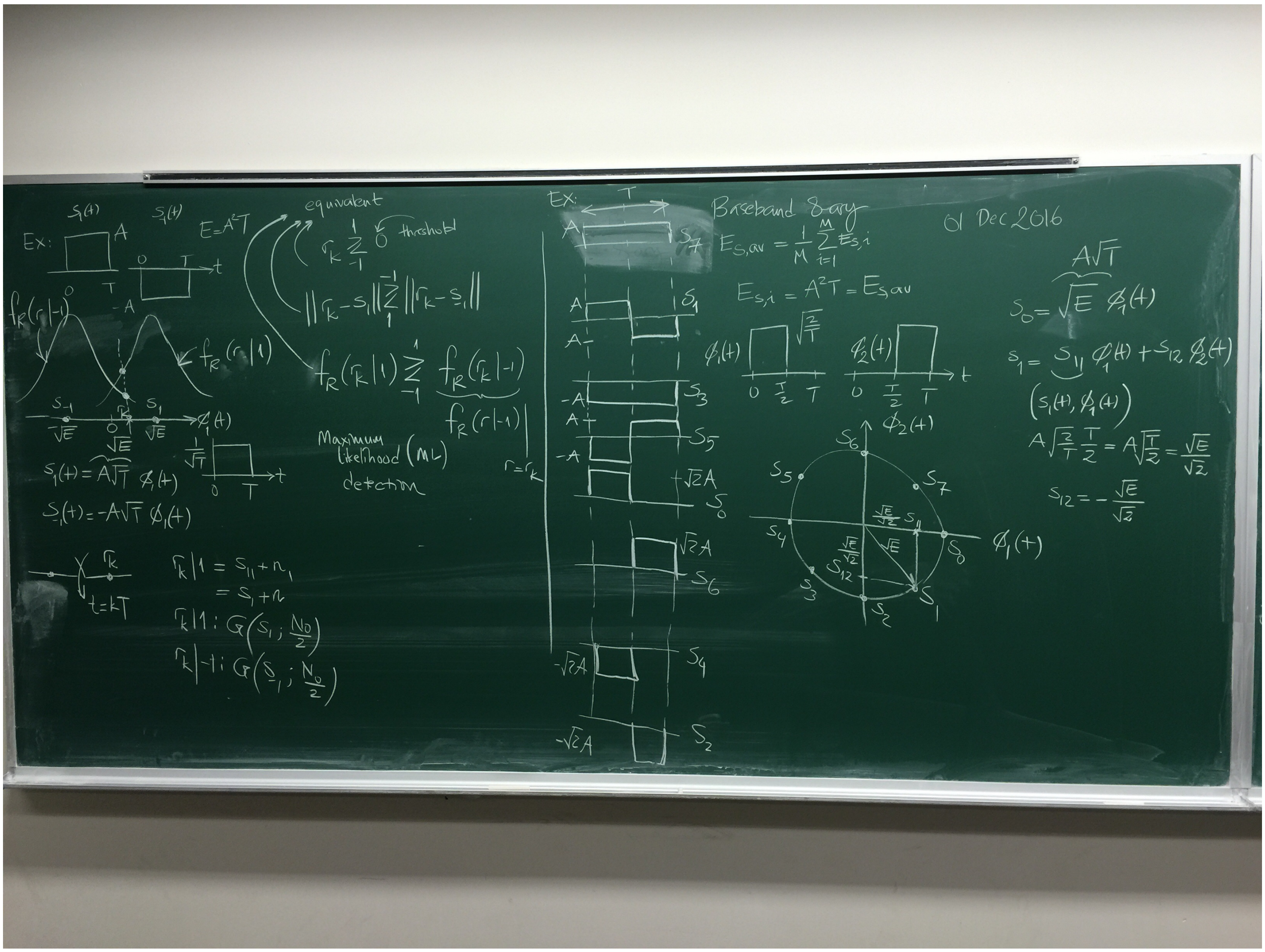

Ex:
E=A²T
equivalent
threshold
Maximum likelihood (ML) detection
Ex:
Baseband 8ary
01 Dec 2016
E_s,av = 1/M Σ_{i=1}^{M} E_s,i
E_s,i = A²T = E_s,av
φ1(t)
φ2(t)
S0 = √E φ1(t)
S1 = S11 φ1(t) + S12 φ2(t)
(S1(t), φ1(t))
A √(2/T) T/2 = A √(T/2) = √E/√2
S12 = − √E/√2
s1(t) = A√T φ1(t)
s−1(t) = −A√T φ1(t)
rk|1 = s11 + n1
= s1 + n
rk|1 : G(s1 ; N0/2)
rk|−1 : G(s−1 ; N0/2)
t=kT

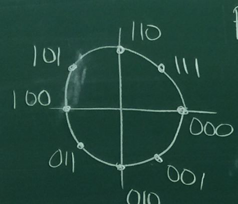
Labeling = Bit-to-symbol mapping

* Natural labeling
(use the symbol index)

s_6 : 110

000 → 111

$P_b \sim P_s$



if 000 is transmitted the most likely erroneous decisions are 111 and 001

Gray Labeling
(Gray coding)

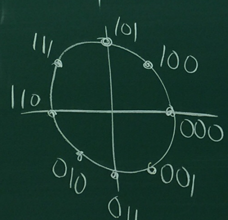
$$P_b \sim \frac{1}{\log_2 M} P_s$$

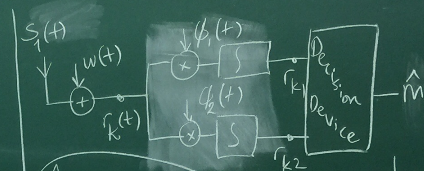
000
001
011
010
110
111
101
100

Ex: $P_s = 10^{-4}$

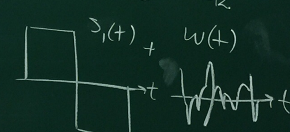
10^6 symbol $= 3\times10^6$ bits

$$P_b = \frac{100}{3\times10^6} = \frac{1}{3}\times10^{-4}$$

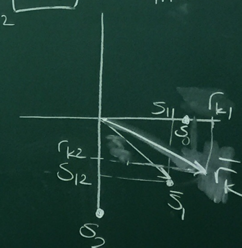




Assume $r_{k1} = 1.5\, s_{11}$
$r_{k2} = 0.8\, s_{12}$



Set $\hat{m} = m_i$ if $\|r - s_j\|$ is minimum for $j = i$



filtering out:
projecting noise on the signal space;
rejecting noise in all other dimensions.

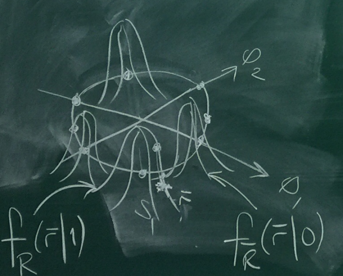
s_1 is transmitted

$f_{R_1}(r_1|1) : G\left(s_{11}; \sigma^2 = \frac{N_0}{2}\right) = \frac{1}{\sqrt{2\pi\frac{N_0}{2}}} e^{-\frac{(r-s_{11})^2}{2\frac{N_0}{2}}}$

$f_{R_2}(r_2|1) : G\left(s_{12}; \frac{N_0}{2}\right)$

$$f_R(\bar{r}|1) = f_{R_1}(r_1|1) \times f_{R_2}(r_2|1)$$

$$= \frac{1}{\pi N_0} e^{-\frac{(r-s_{11})^2 + (r-s_{12})^2}{N_0}}$$



Set $\hat{m} = m_i$ if

$f_R(\bar{r}_k|j)$ is maximum for $j=i$

Observed vector

Optimum Decision Rule

Set $\hat{m} = m_i$ if

$P(m_j \text{ is sent} \mid \bar{r})$ is maximized for $j=i$

Maximum A Posteriori (MAP) rule

Bayes' Rule

$$P(A|B)\cdot P(B) = P(AB)$$
$$P(B|A)\cdot P(A) = P(AB)$$

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(m_j|\bar{r}) = \frac{P(\bar{r}|m_j)\, P_{m_j}}{P(\bar{r})} \rightarrow \frac{f(\bar{r}|m_j)\, \overbrace{P_{m_j}}^{\frac{1}{M}}}{\underbrace{f(\bar{r})}_{\text{constant}}}$$

Set $\hat{m} = m_i$ if

$f(\bar{r}|m_j)$ is maximized for $j=i$

Maximum Likelihood principle

$MAP = ML$

if all symbols are equally-likely

Set $\hat{m} = m_i$ if

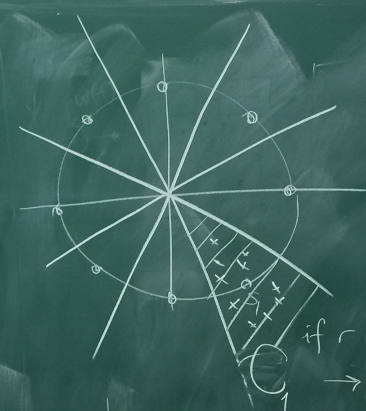
$\| r - s_j \|$ is minimized for $j=i$

minimum distance rule

log likelihood

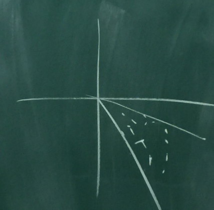
$$\underbrace{-\frac{1}{N_0}\sum_{l=1}^{N}(r - s_{jl})^2}_{\text{maximizing}}$$

$$= \text{minimizing } \sum (r - s_{jl})^2$$



if r is in region C_1

$\rightarrow \hat{m} = m_1$



Netflix example

"learning"