Minimising BER by maximizing Noise Ratio [Discussed Before]

Minimising transmission BW:

Assume \( R = R_0 \) bits/sec.

Consider \( h_{TX}(t) \); bit stream 1, 1, -1.

**TX I.**

\[ T_0 = \frac{1}{R_0} \]

\( T_0 = \frac{T_0}{5} \)

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Assumed for this example

\[ BW = \frac{1}{T_0} = R_0 (Hz) \]

\[ BW \sim 5R_0 \ HZ \]

Bit Energy: \( E_b, I = \frac{1}{5} E_b, II \).

Discussion: TX I: has synchronization advantage for long stream of data.

- has lower SNR as a disadvantage because bit energy less.

Compensation: More TX power

- More expensive power amplifier.

**TX II.**

\[ T_0 = \frac{1}{R_0} \]

**TX III.**

\[ T_0 = \frac{1}{R_0} \]

\[ BW \sim \frac{R_0}{2} (Hz) \]

Problem: ISI

- Noise Aside.
- \( w(t) \) (SMF: irrelevant)
- no restriction on \( h_{TX}, h_{CH}, h_{RX} \).
- Many PAM.
\[
\Sigma_k a_k S(t-kT) h_{tx}(t) h_{ch}(t) h_{rx}(t) \xrightarrow{KT} \{ a_k \}
\]

\[
p(t) = h_{tx}(t) * h_{ch}(t) * h_{rx}(t).
\]

\[
P(f) = h_{tx}(f) * h_{ch}(f) * h_{rx}(f).
\]

**Find structure of \( p(t) \) such that**

- \( \star \) no ISI
- \( \star \) min BW

**Ex 1:**

Bit Stream: \( a_0 \)

**Ex 2:**

\[
y(t) = a_0 p(t) + a_1 p(t-T) + a_2 p(t-2T) + \cdots
\]

\[
y(\omega) = y(t=0) \text{ desired } = a_0 p(0) + a_1 p(-T) + a_2 p(-2T) + \cdots
\]

For no ISI:

\[
p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = kT \end{cases}
\]

Need 1 at origin, \( a \) at other \( T \) intervals.

Infinitely many options.
\[ \Sigma_k (t-kT) \times p(t) = s(t) \]

Impulse
Taking Fourier transform
\[ \Sigma_k (t-kT) \times p(t) = s(t) \]
\[ \mathcal{F} \left\{ \Sigma_k (t-kT) \times p(f) \right\} = 1 \]
\[ \frac{1}{T} \Sigma s(f-k/T) \times p(f) = 1 \]
\[ \Sigma p(f-k/T) = T \] no ISI condition in freq domain

Nyquist criteria for no ISI:
\[ \Sigma_k p(f-k/T) = T = \ldots + p(f+1/T) + p(f) + p(f-1/T) + \ldots = T \]

Ex. 1:

\[ p(f) \]

Which \( p(f) \) results in minimum

Minimum Bandwidth: \( p(f) \)

Solution

\[ -2T \quad 0 \quad 2T \]

\[ -1/T \quad 0 \quad 1/T \]

BW?

\[ p(f+1/T) \]

\[ p(f) \]

\[ p(f-1/T) \]

Delta is not minimum BW \( \leq \Sigma p(f-k/T) \)
Minimum BW \( P(t) \)

\[ \text{sol.} \quad \text{no ISI} \]

\[ -2T \quad 0 \quad 2T \]

**Revisit:** What if BW is much narrower than mentioned before?

\[ P(f) \]

\[ -\frac{1}{2T} \quad \frac{1}{2T} \]

**Gap \( \leq P \left( f - \frac{k}{T} \right) = ? \) T.**

Gaps will occur

Not best solution

Example:

\[ p(t) = \text{sinc}^2 \left( \frac{t}{T} \right) \]

no ISI?

\[ 1 \]

\[ -2T \quad 0 \quad 2T \]

**Spectral Efficiency:**

\[ SE: \mu \]

\[ K_{\text{max}} = \frac{R}{\text{BW} \cdot H_3} \text{ sym/sec} \]

\[ \frac{R}{2} \text{ H}_3 \]

\[ p(f) \cos(2\pi f_c T) \]

\[ H_{\text{max}} = \frac{1}{\text{sym/sec/H}_3} \text{ [Bandpass]} \]

\[ P_{\text{max}} = H_{\text{max}} \times \text{BW} \]

Ex: Telus lease 5MHz of BW

\[ P_{\text{max}} = 5 \text{ Msym/sec} \]