1/11/2016 SYSC 4600

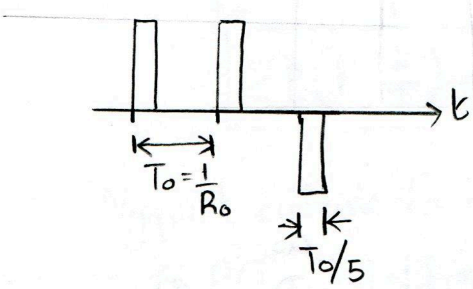
Minimising BER by maximizing Noise Ratio [Discussed Before]

Minimising transmission BW:

Assume $R = R_0$ bits/sec.

Consider $h_{TX}(t)$; bit stream 1, 1, -1.

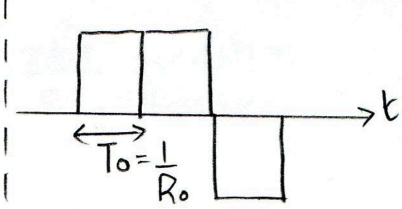
TX I.



Assumed for this example

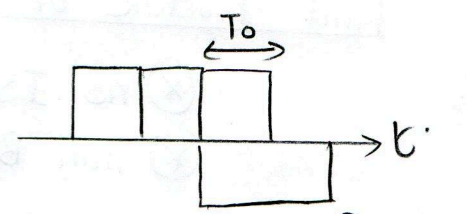
$BW \sim 5R_0$ Hz

TX II.



$BW \cong \frac{1}{T_0} \cong R_0$ (Hz)

TX III



$BW \sim \frac{R_0}{2}$ (Hz)

Problem: ISI.

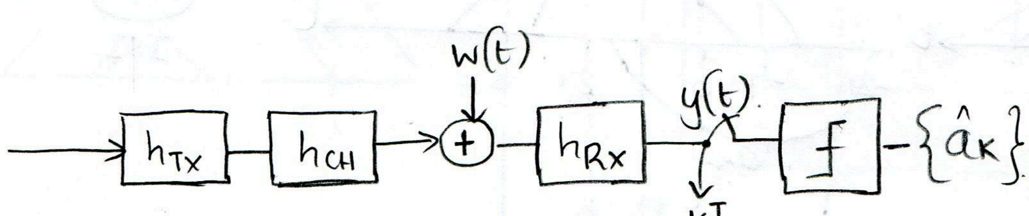
Bit Energy: $E_{b,I} = \frac{1}{5} E_{b,II}$

Discussion: TX I: has synchronization advantage for long stream of data.

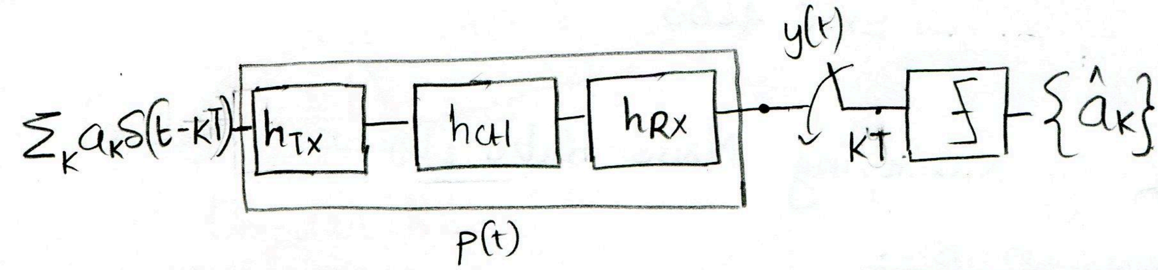
: has lower SNR as a disadvantage because bit energy less.

Compensation: More TX power

More expensive power amplifier.



- Noise Aside
- w(t) (→MF: irrelevant)
- no restriction on h_{TX}, h_{CH}, h_{RX}.
- Mary PAM.



$$p(t) = h_{TX}(t) * h_{CH}(t) * h_{RX}(t)$$

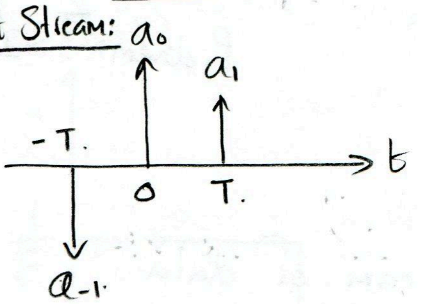
$$P(f) = h_{TX}(f) * h_{CH}(f) * h_{RX}(f)$$

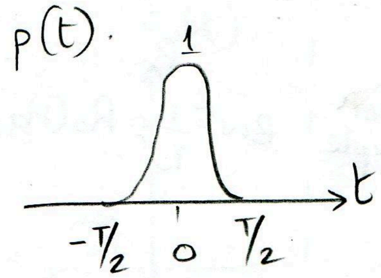
Find structure of p(t) such that

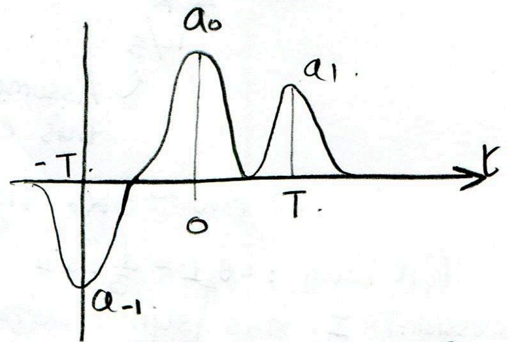
- no ISI
- min BW

Ex 1:

Bit Stream:

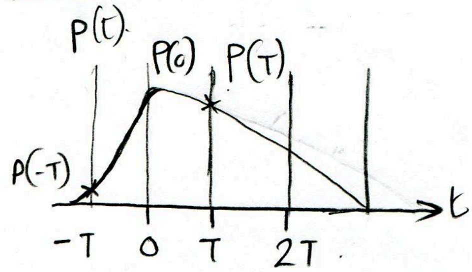


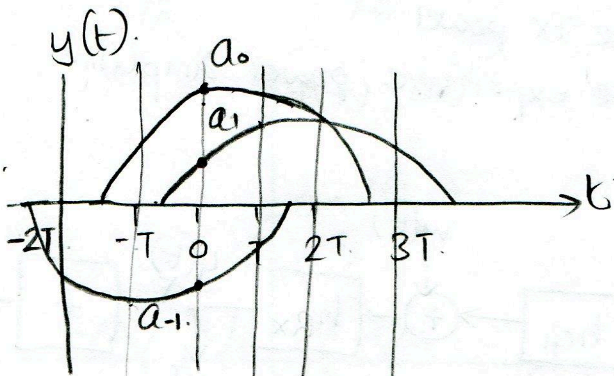




$y(KT) = \ldots, a_{-1}, a_0, a_1, \ldots$

Ex 2:





$y(0) = y(t=0)$ desired

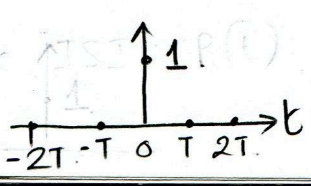
$= \underbrace{a_{-1}P(T)}_{\text{ISI}} + a_0 P(0) + \underbrace{a_1 P(-T)}_{\text{ISI}}$

$$y(0) = \sum a_K P(-KT)$$

$$= \underbrace{\ldots\ a_{-2}P(2T) + a_{-1}P(T)}_{\text{ISI}} + \overset{\text{Desired}}{a_0 P(0)} + \underbrace{a_1 P(-T) + a_2 P(-2T) + \ldots}_{\text{ISI}}$$

For no ISI:

$$\boxed{p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = KT \end{cases}}$$



Need 1 at origin & 0 at other T intervals

Infinetely many options.

$$\sum_K (t-KT) * p(t) = \delta(t)$$

Impulse string.

Taking Fourier transform

$$\Sigma_K (t-KT) * p(t) = \delta(t)$$

$$FT\{\Sigma_K (t-KT)\} * P(f) = 1$$

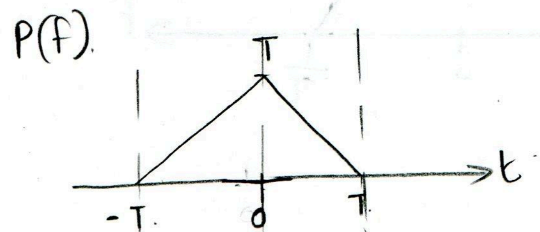
$$\frac{1}{T}\Sigma \delta\left(f-\frac{K}{T}\right) * P(f) = 1$$

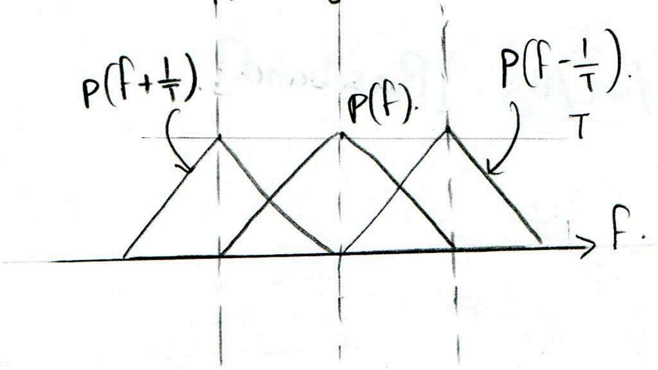
$$\boxed{\Sigma P\left(f-\frac{K}{T}\right) = T}$$ no ISI condition in freq domain.

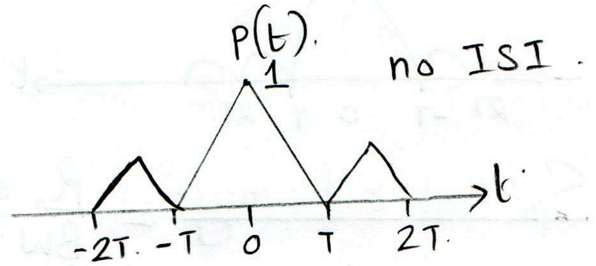
Nyquist criteria for no ISI:

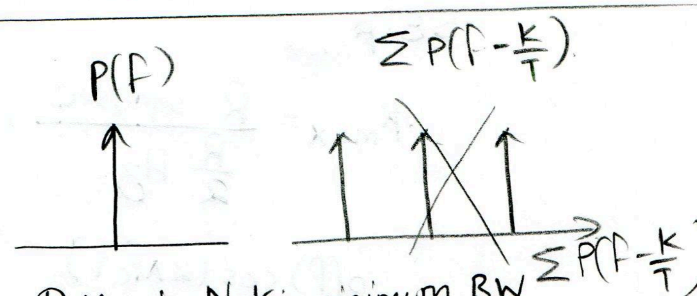
$$\sum_K P\left(f-\frac{K}{T}\right) = T = \dots + P\left(f+\frac{1}{T}\right) + P(f) + P\left(f-\frac{1}{T}\right) + \dots = T$$

Ex 1:





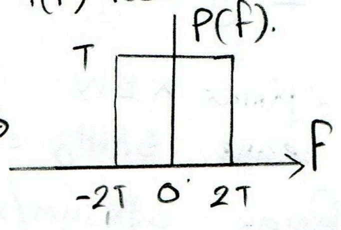


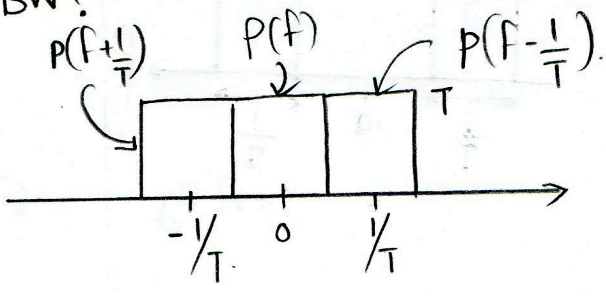


Delta is Not minimum BW

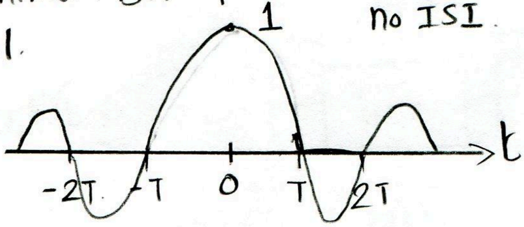
Which P(f) results in minimum BW?

Minimum Bandwidth Solution :→

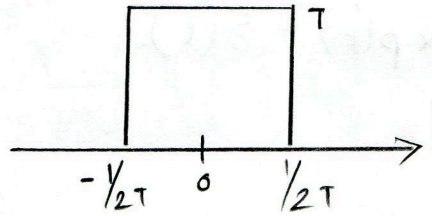




Minimum BW $p(t)$ no ISI sol.

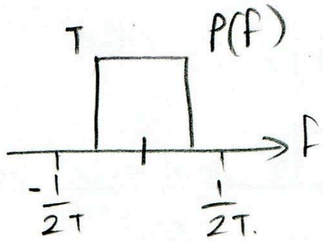


$P(f)$

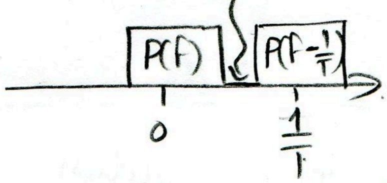


$BW = \frac{1}{2T} = \frac{R}{2}$ Hz

Revisit: What if BW is much narrower than mentioned before?



Gap. $\sum P(f - \frac{k}{T}) = ?\, T$



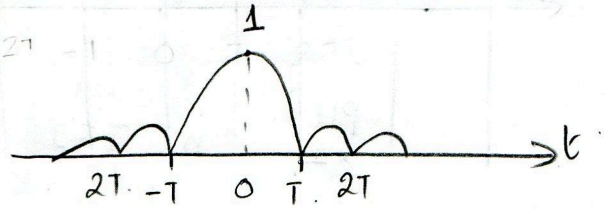
Gaps will occur

∴ Not best solution

Example:

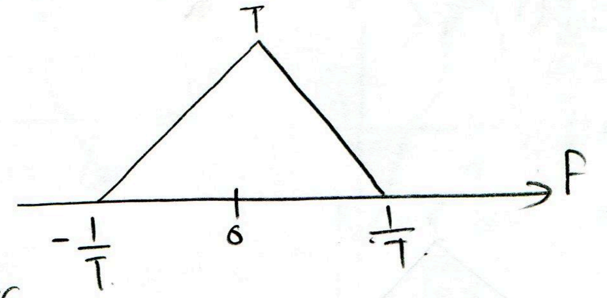
$p(t) = \text{sinc}^2(t/T)$.

no ISI?



no ISI ✓

$P(f)$.

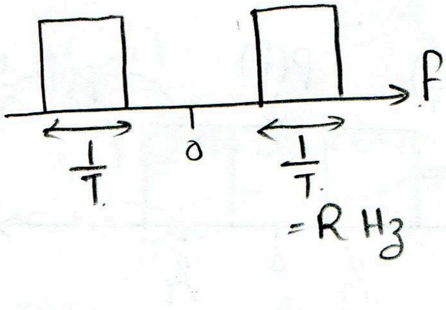


Spectral Efficiency: $\triangleq \frac{R}{BW} \frac{\text{sym/sec}}{\text{Hz}}$

SE: μ_{max}

$$\mu_{max} = \frac{R \text{ sym/sec}}{\frac{R}{2} \text{ Hz}} = 2 \text{ sym/sec/Hz} \quad \text{[Baseband]}$$

$p(f) \cos(2\pi f_c T)$.



$\mu_{max} = 1$ sym/sec/Hz [Bandpass]

$R_{max} = \mu_{max} \times BW$.

Ex: Telus leases 5 MHz of BW

$R_{max} = 5$ Msym/sec.