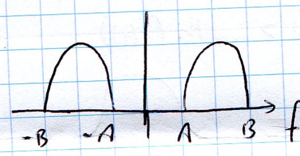
SYSC 4600 — 13-Sept-2016

website = sce.carleton.ca/courses/sysc-4600/f16/

* 3 ways to increase rate.
  - Higher bandwidth.
  - Increase SNR
  - Increase # of antennas
* SNR and rate ratio is logarithmic. you have increase SNR a lot to get a little bit of rate.
*

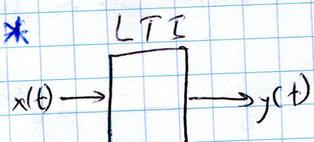
15-Sept-2016

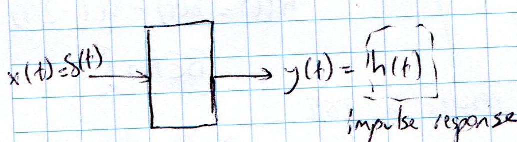
* Calculate the bandwidth always from positive side



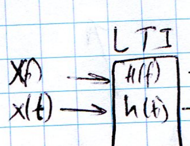
$\Longrightarrow BW = B - A$

* The worst case scenario for binary system is 0.5 because if it is 1 you can just solve it by putting an inverter before that.
* One question will be from the convolution on the test!!! *
*





proof it as a homework
why?

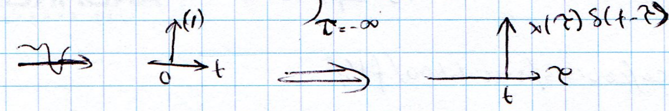


$Y(f) = X(f)H(f)$ ⟹ simpler

$y(t) = x(t) * h(t)$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

* $x(t) * \delta(t) \triangleq \int_{\tau=-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$



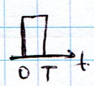
– Point by point multiplation of both graphs.

* Delta function is the identity function with respect to convolution

Ex: $x(t) * \delta(t-t_0) = x(t-t_0)$

$x(t-t_0) * \delta(t) = x(t-t_0)$

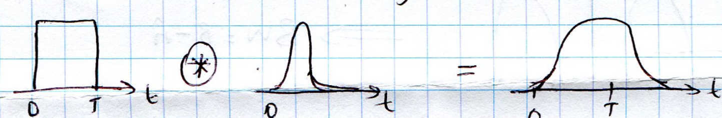
Ex: $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

1. Case  $\underbrace{h(t)=\delta(t)}_{\text{short circuit}}$ $y(t) = x(t)$

2. Case $h(t) \neq \delta(t)$ $y(t) = x(t) * h(t)$

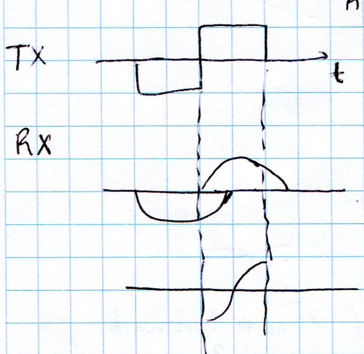
$$\text{width}(y(t)) = \text{with}(x(t)) + \overbrace{\text{width}(h(t))}^{>0}$$

$$\therefore \text{width}(y(t)) > \text{width}(x(t))$$



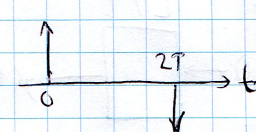
This is ISI (inter-symbol-interference) and this has much more influence than the noise. To cure it use equalizer.

---- 0 1 ---- $R = \frac{1}{T}$ bits/sec



pulse broadenig results in ISI

Ex: $h(t) = 2$-path wireless channel

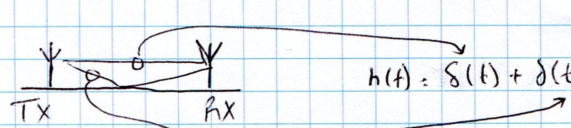


$h(t) = \delta(t) - \delta(t-2T)$

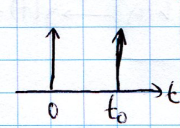
BER = ?

* Chanel is independent from rate.

Ex: 2 path wireless channel

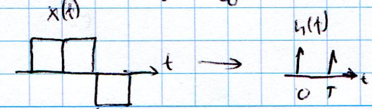


$h(t) = \delta(t) + \delta(t-t_0)$



bit direction

Assume $t_0 = T$

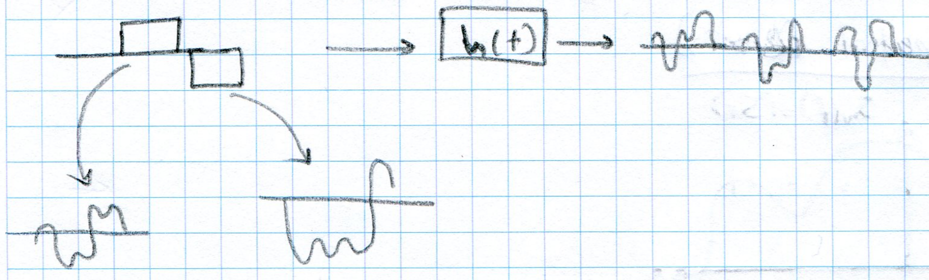


$BER = \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{4}$

SYSC 4600 — 20-Sep-2016

ARCHROMA LIFE ENHANCED

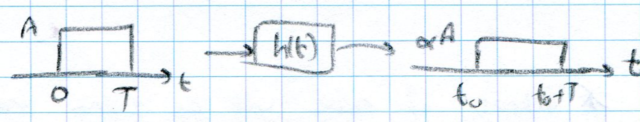
Major Impairments in Communication Systems



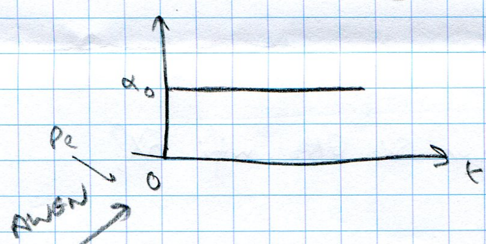
ideal channel;

$h(t) = \alpha\, \delta(t - t_0)$ — α: attenuation, t_0: propagation delay

ideal channel + noise = AWGN

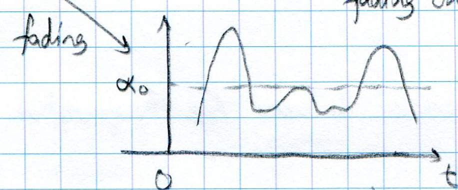


if $\alpha \ll 1$ (very high path loss)



if $\alpha = \alpha_0 \rightarrow$ SNR: very high

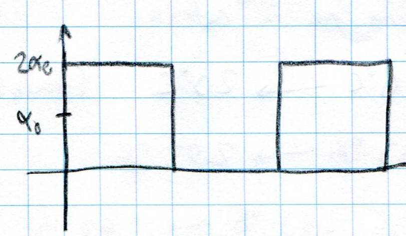
fading channel: $\alpha\, \delta(t - t_0)$ — RV



SNR: very low

Fading Channel

special case



$$Pe = \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$

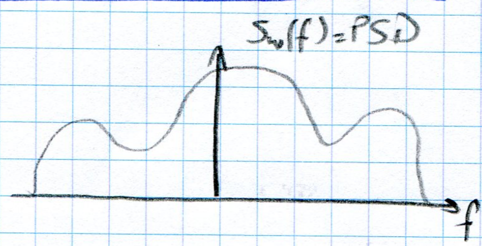
$$= 0.25$$

→ Layer 1 = Physical

→ Layer 2 = link layer (radio resourse management, ...)

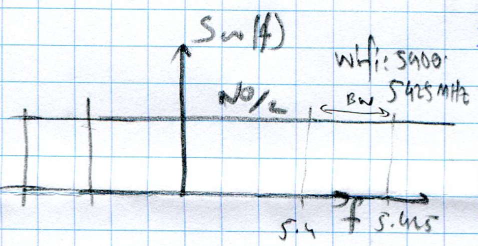
* $SNR = \frac{P_s}{P_n}$ → noise power in your signal BW

PSD (Power Spectral Density)



Q) How much power does the process have between f_1 # and f_2 #?

$$P_{f_1,f_2} = 2\int_{f_1}^{f_2} S(f)\,df$$



* Intensity of noise will be same and it is infinite in this case.

Q) How much noise power is there in your 5.4 GHz wifi RX?

$$2 \times \frac{N_o}{2} B = N_o B$$

ecause of
e side

→ $N_o = kT$

↳ as degrees increase N_o also increase however the temperature is in kelvin no in celcius

10°C → 20°C (doubled? N_o)

283 °K → 293 °K

N_o = 283 °K → 293 °K only ↑~%3

Noise Power

$$P_N = \overbrace{kT}^{N_0}BF \quad \text{(Watts)} \rightarrow \text{linear}$$

$$k + T + B + F \quad \rightarrow \text{logaritmic} \quad \text{(Communication Engineers works with this)}$$

Eg : $N_0 = kT$

$$= 1.38 \times 10^{-23} \times 293 = 4.04 \times 10^{-21} \text{ (linear)}$$

$$= \log(4.04 \times 10^{-21}) \times 10$$

$$= -203.9 \text{ dBW/Hz} \quad \text{(logarithmic)}$$

$$= -174 \text{ dBm/Hz}$$

Q01 1 MG ~~Bits~~/s $\rightarrow$ SNR = 15 dB

10 MG Bit/s $\rightarrow$ SNR = ?

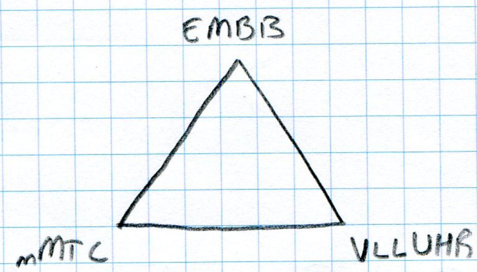
Rate increase 10 times so BW increase 10 times, noise power increase 10 times and SNR decrease 10 times

which is SNR = 5 dB

SYSC 4600 22-Sep-2016

Fundamental Dynamics of Digital Communication

ITU-R $\begin{cases} \text{IMT - Advanced} & 4G \\ \text{IMT - 2020} & 5G \end{cases}$

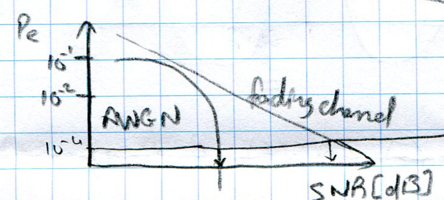


5G Phase 1 — release 15 Mar 2018

5G Phase 2 — release 16 Dec 2019

2020 →

* $P_e \approx e^{-SNR}$ → Increase SNR to decrease P_e (Probability of error or bit error rate (BER))



@ 10^{-5} difference is 35 dB between fading and AWGN

$P_e \approx SNR^{-1}$

→ QoS :
- Reliability (i.e, min P_e)
- Rate
- Latency
- energy efficiency

(conflict: Reliability ↕ Rate)

* Increasing SNR will increase the bit rate.

→ M-QAM

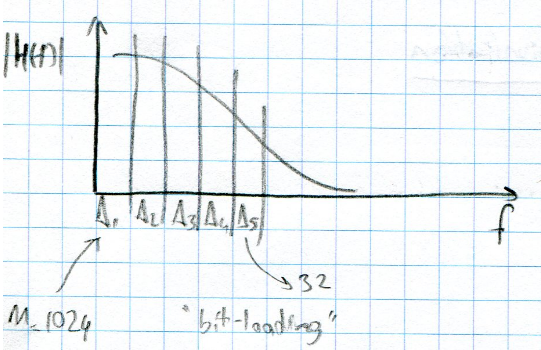
M-ar

binary = 2-ary → 1 sym = 1 bit

4-ary → 1 sym = 2 bit

M-ary → 1 sym = $\log_2 M$

xDSL

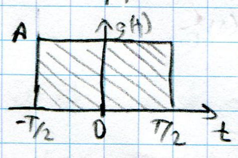


* As frequency increase the SNR decreases

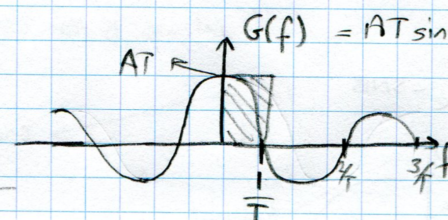
Rate Vs Bandwidth

→ For R bits/sec , how much BW do I need for binary?

$T = \frac{1}{R}$ sec



Fourier Transform



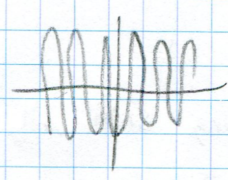
$G(f) = AT\,\mathrm{sinc}(ft)$

Power = A^2 Watts

modulation

$$g(t) = \int G(f)\, e^{j2\pi ft}\, df$$

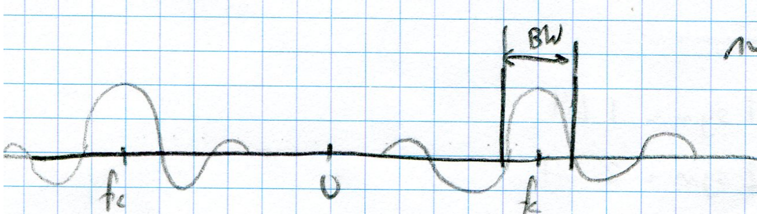
$$G(f) = \int g(t)\, e^{-j2\pi ft}\, dt$$



$$\underbrace{G(f=0)}_{\text{zero-value}} = \int g(t)\, dt$$

Fourier Transform

null-BW $= \frac{1}{T}$ (always read from the +ve side)



null to null BW = $\frac{2}{T}$

* It is obvious that modulation doubles the BW.

→ Linear relation between R and BW ,

with rect pulses (baseband) BW=R

SYSC 4600 - PA 23-Sep-2016

TA

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dB Conversion

dBW ± dB = dBW → Transmit Power

→ dB is unitless

→ SNR is unitless (Watt/Watt = Unitless)

dBm ± dB = dBm

dBW - dBW = dB

~~dBW + dBW =~~

dBW = dBm - 30 ⟹ important for course

Quiz 1 - 2015

Question 2) → AWGN channel prefer no shape change in the signal from transmitter to receiver.

$SNR = \frac{P_s}{P_N}$ [unitless]

Given

SNR = 6 dB

$N_0 = -174$ dBm/Hz

BW = 1 MHz → 60 dBHz ⟹ $10\log_{10}(x)$

F = 10 dB

$P_s = ?$

SNR[dB] = $P_s - P_N$ [dB]

Noise Power ⇐ $P_N = N_0 + B + F$ [dB]

$= -174 \frac{dBm}{Hz} + 60\ dBHz + 10\ dB$

$= -104$ dBm $= -134$ dB

$= 10^{-134}$ Watts

Quiz 2 - 2015 - Question 2

SNR = 8.4 dB

$N_0 = -174$ dBm/Hz

B = 200 kHz

F = 8 dB

SNR = $P_s - P_N$ [dB] ; P_s in Watts = ?

SNR from graph → 10^{-4} goes to 8.3

Assignment 1 2016, Question 5

$X = \{1, 4, -3, -2, 5\}$

$E[X] = \frac{1}{5} + \frac{4}{5} - \frac{3}{5} - \frac{2}{5} + \frac{5}{5} = 1$

$Var[X] = E[X^2] - E^2[X]$

$= \{1^2 + 4^2 + (-3)^2 + (-2)^2 + 5^2\} \cdot \frac{1}{5} - 1^2 = 10$

Question 7

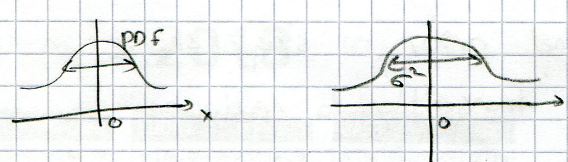
$X \sim U[\overset{a}{0}, \overset{b}{4}]$

$E[X] = \frac{a+b}{2}$, $Var[X] = \frac{(b-a)^2}{12}$

$Prob(2 < X < 3) = \int_2^3 x \cdot \underbrace{F_x(x')}_{PDF} dx$

* PDF is important for bit error rate!

PDF - CDF (STAT)

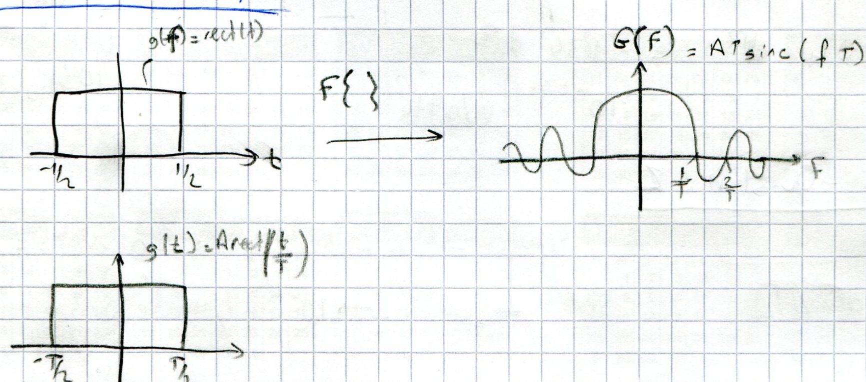


$X = Y + Z$

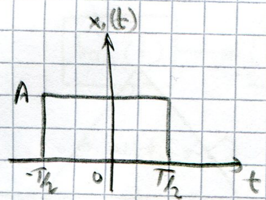
$Y \sim N(\mu_y, \sigma^2_y)$, $Z \sim N(\mu_z, \sigma^2_z)$

$X \sim N(\mu_y + \mu_z, \sigma^2_y + \sigma^2_z)$

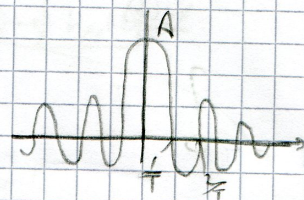
Fourier Transform



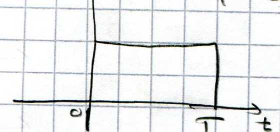
Assignment 2 - Question 1



$\longrightarrow$

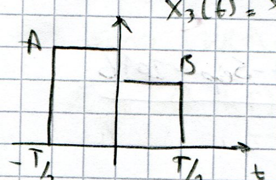


$x_2(t) = x_1\left(t - \frac{T}{2}\right)$



$\longrightarrow$ $X(f)\, e^{-j 2\pi f T/2}$

$x_3(t) = x\left(2t - \frac{T}{2}\right) + \frac{B}{A} x_1\left(2t + \frac{T}{2}\right)$



Fourier Transfer Identities

$\delta(t) = 1$

$\delta(t - t_0) \rightarrow 1 \cdot e^{-j 2\pi f t_0}$

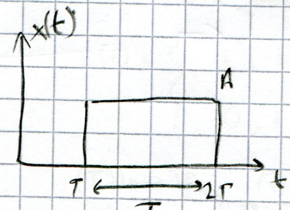
$f(t - t_0) \rightarrow F(f)\, e^{-j 2\pi f t_0}$

$f(at) \rightarrow \frac{1}{|a|} F\left(\frac{f}{a}\right)$

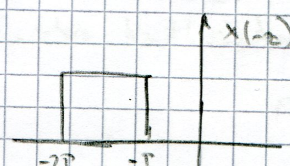
$f(t) * g(t) \rightarrow F(f) \cdot G(f)$

$f(t) + g(t) = F(f) + G(f)$

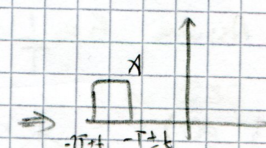
Quiz 2 - Question 1

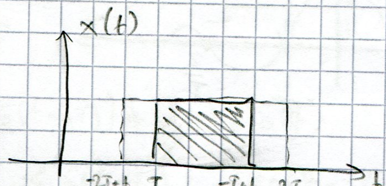


a) $y(t) = x(t) * x(t)$



$\Rightarrow$



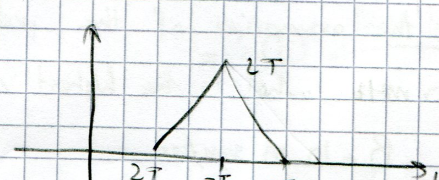


$0 \leftarrow t < 2T$

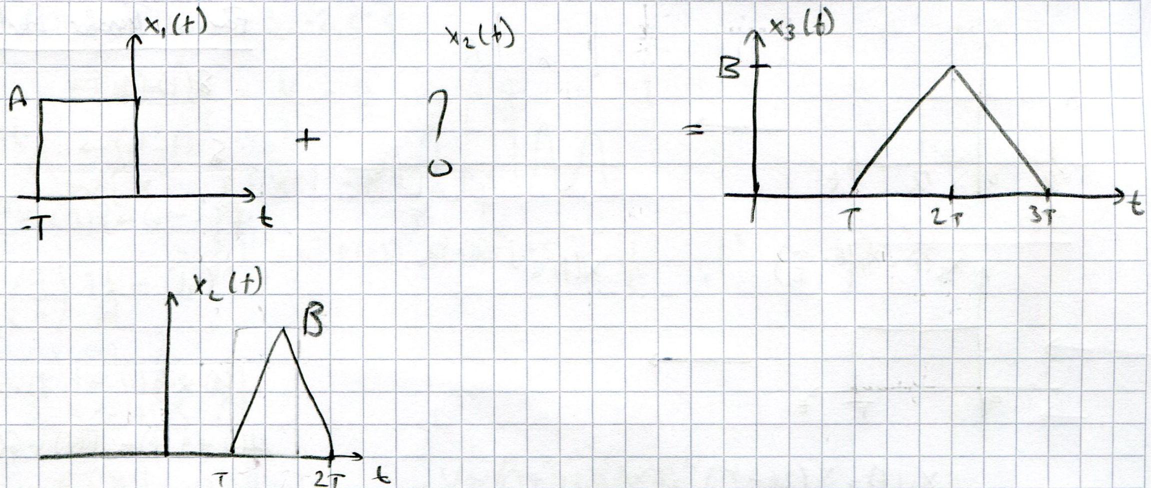
$A^2(\overbrace{-T + t - T}^{t-2T}) \leftarrow 3T > t > 2T$

$A^2(2T + 2T - t) \leftarrow 4T > t > 3T$

$0 \leftarrow 4T < t$



Assigment 2 - Question 3



Lecture

27-Sep-2016

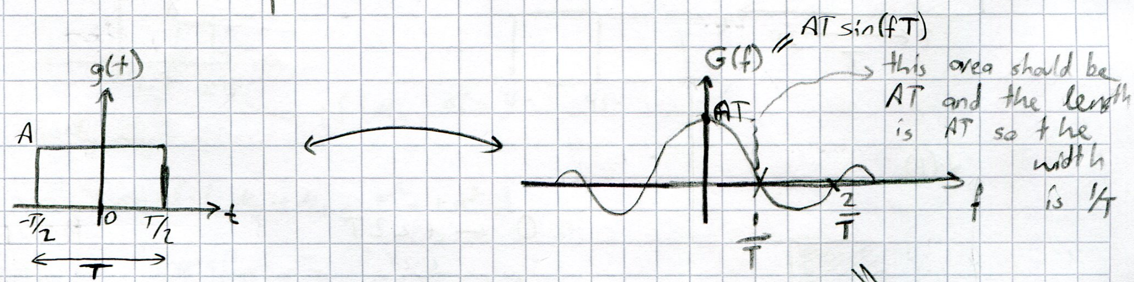
Q) Where does the rate come from?

A) Most fundamental rate is coming from SNR -

Q) i) How fast can I send pulses (symbols)? In 1 Hz what is the max sym/sec?

Q) ii) How many bits can I embed in a symbol?

A) i) Assume $R = \frac{1}{T}$ sym/sec



* Area under curve of g(t) is zero point in G(f)

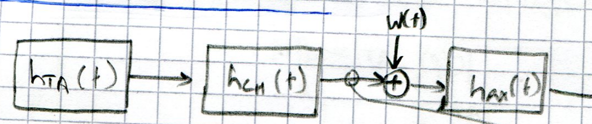
null $BW = \frac{1}{T} = R$ Hz (baseband)

* BW depends on pulse shape.
* Under the assumptions of the preceeding example; If I am green, $B = 10$ MHz, what is the highest rate achievable?

  $R = 10$ M sym/sec

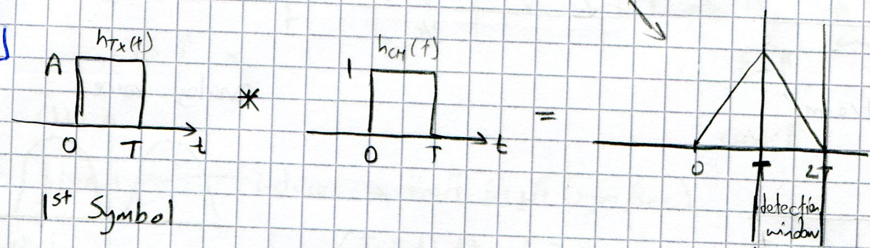
* If you transmit carelessly, then will run into ISI.

ISI-free Transmission



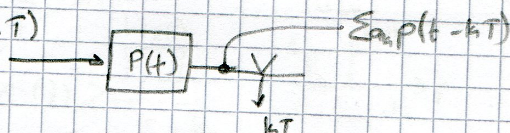
• $w(t) = 0$

Ex)



• For a given $h_{CH}(t)$, design $h_{TX}(t)$ and $h_{RX}(t)$ so that no ISI.

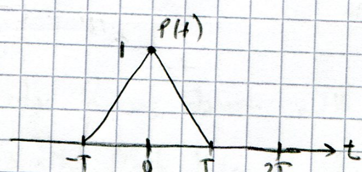
→ since $w(t) = 0$

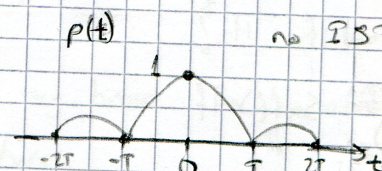


$$p(t) = h_{TX}(t) * h_{CH}(t) * h_{RX}(t)$$

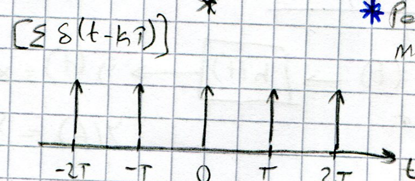
$$P(f) = H_{TX}(f)\, H_{CH}(f)\, H_{RX}(f)$$

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = kT, \text{ where } k = \pm 1, \pm 2, \dots \end{cases}$$





$*$



$*$ Point by point multiplication

$$\left[\Sigma \delta(t-kT)\right] * p(t) = \delta(t)$$

$$\left[\frac{1}{T} \Sigma \delta\left(f - \frac{k}{T}\right)\right] * P(f) = 1$$

$$\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) = T \quad \text{(No ISI)}$$

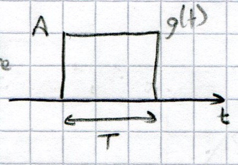
$= \delta(t) \xleftrightarrow{\text{fourier transform}} 1$

Lecture

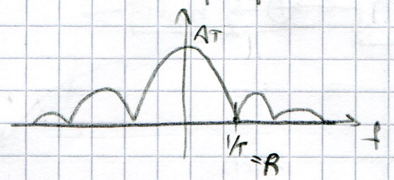
29-Sep-2016

BW Calculation

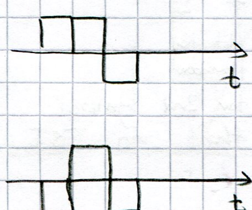
determine



$|G(f)| = AT|\text{sinc}(fT)|$



"random bit stream"

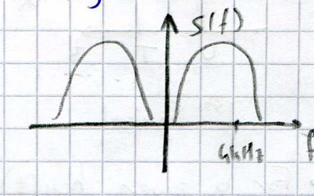


$$x(t) = \sum \underset{\pm 1}{a_k}\, g(t - kT - \underbrace{T_0})$$

$$\rightarrow S_x(f) = \frac{1}{T}|G(f)|^2$$

$$S_x(f) \propto |G(f)|^2$$

analog voice



$FT\{x_1(t)\} = X_1(f)$

$FT\{x_2(t)\} = X_2(f)$

if $x_1(t) \neq x_2(t) \longrightarrow X_1(f) \neq X_2(f)$

* even if condition is not satisfied.

* conditions a_k unrelated $E[a_k] = 0$

- determine signals $\longrightarrow$ FT
- random signal $\longrightarrow$ PSD

Q) How much power does the process $x(t)$ have between frequencies f_1 and f_2 Hz?

$$2\int_{f_1}^{f_2} S_x(f)\, df$$

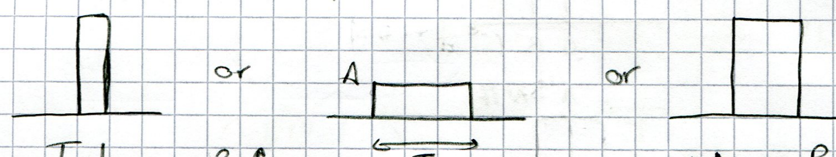
Linear Systems

$\underset{\text{det}}{x(t)} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$

$Y(f) = X(f)H(f)$

$\underset{\text{random}}{x(t)}$ $\quad S_x(f) = |H(f)|^2 S_x(f)$

----, -1, +1, 1, ---- pulse shape : rectangular



$T \downarrow \Rightarrow R \uparrow$ ✓

$B \uparrow$ ×

$A \uparrow \Rightarrow P_{Tx} \uparrow \rightarrow P_{Rx} \uparrow \Rightarrow SNR \uparrow$

$E \uparrow$ ↑ reliability rate ↑

cost ↑

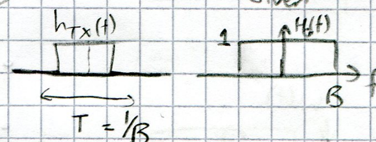
* B : given ⟶ induces a limit in R (sym/sec)

⟶

$2a_k h_{Tx}(t-kT)$

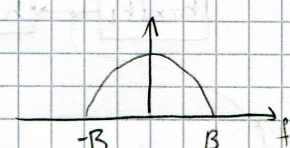
$$\sum a_k \delta(t-kT) \longrightarrow \boxed{h_{Tx}(t)} \xrightarrow{A} \boxed{h_{CH}(f)} \xrightarrow{A'} \oplus \leftarrow w(t)$$

Given



PSD at B

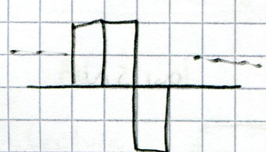
$S_{A'}(f) \triangleq S_A(f)\,|H_{CH}(f)|^2$

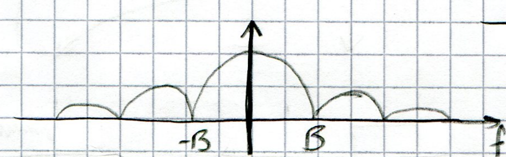


PSD at A

$S_A(f) \propto |H_{Tx}(f)|^2$

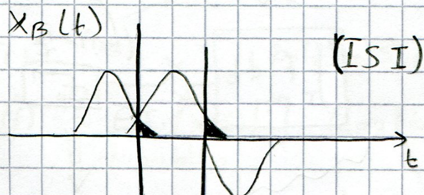
$x_A(t)$



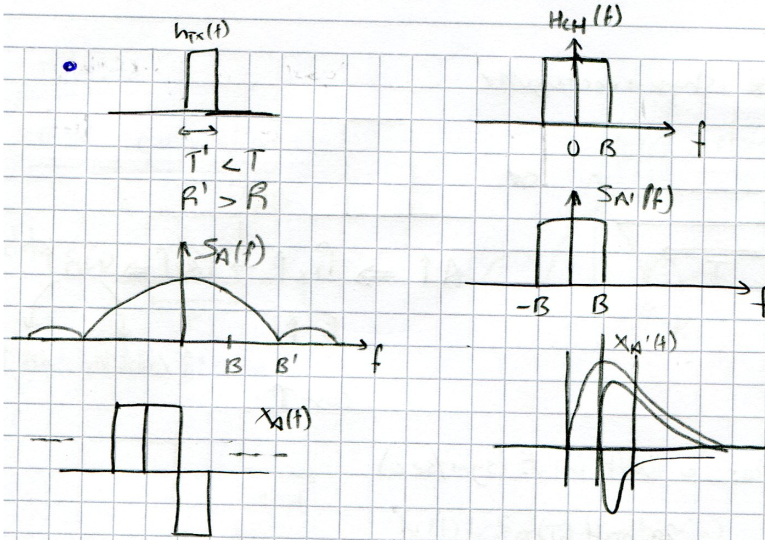


* Having edge in one domain will make very broad in the other domain, such as the first thing in this lecture.

$x_B(t)$



(ISI) (mild)



high ISI

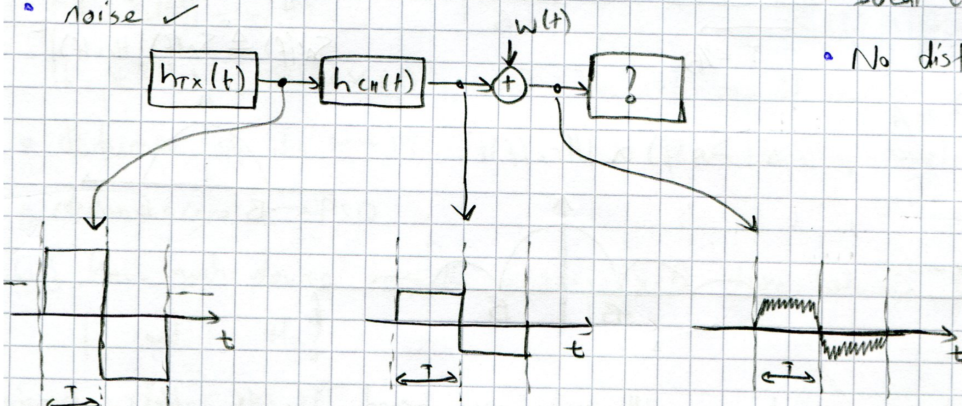
BER ↑

low reliability.

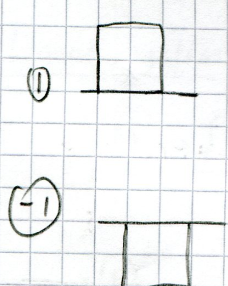
Introduction to Detection/Estimation Theory

- Ignore ISI
- noise ✓

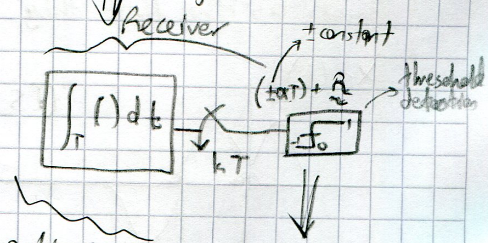
- Ideal Channel
- No distortion



low SNR



integrate and down circuit

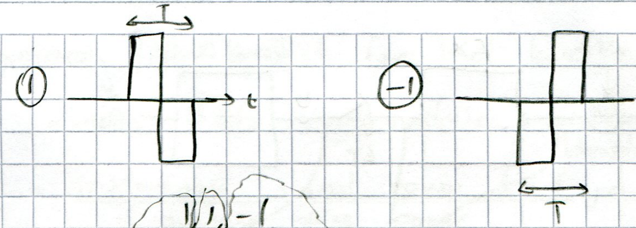


optimal receiver with respect to reliability.

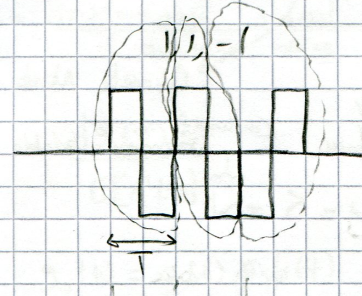
* if it is less than 0 it says "-1" and if it is bigger than 0 it say "+1".

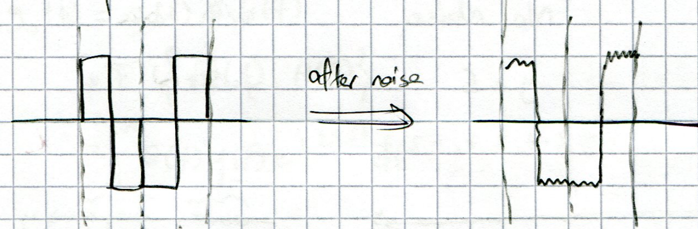
⟹ reliability means (min BER)

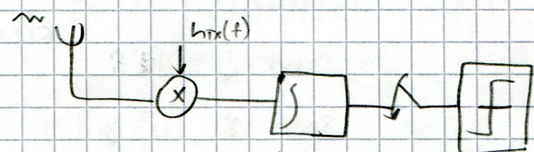
*

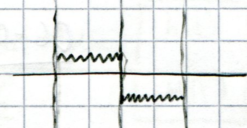


"good for synchronization"
(but using twice BW)



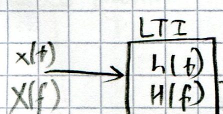






Lecture

04-Oct-2016



$y(t) = x(t) * h(t)$
$Y(f) = X(f)\,H(f)$

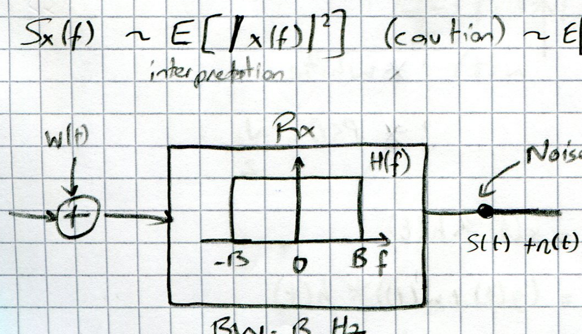
Random $\underset{\sim}{x}(t)$

$S_y(f) = S_x(f)|H(f)|^2$

$S_x(f) \sim E\left[|x(f)|^2\right]$ (caution) $\sim E\left[\underset{\sim}{x}^2(t)\right]$

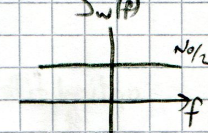
interpretation

*Power is square of a signal.

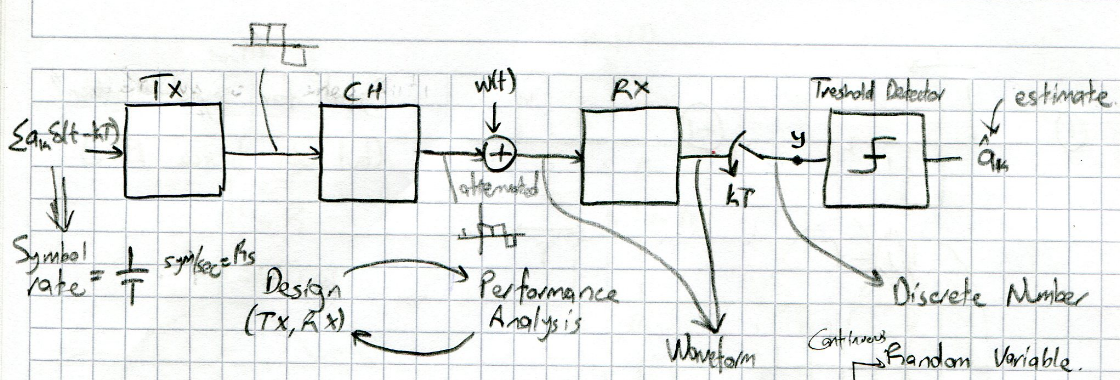


Noise Power $\triangleq \int_{-\infty}^{\infty} S_N(f)\,df$

$\int_{-\infty}^{\infty} S_w(f)|H(f)|^2\,df$



$= \frac{N_0}{2}\int |H(f)|^2\,df = \frac{N_0}{2}\int_{-B}^{B} 1\,.df$



$y = S + N$

No Noise

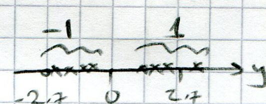
$y = S$

2.7, 2.7, -2.7

$P_e = 0$

with Noise

$y = S + N$



SNR ↓ , P_e ↑

SNR ↑ , P_e ↓

* P_e = BER

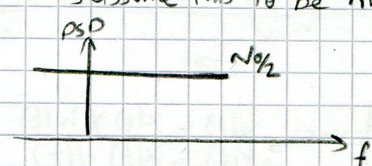
Lecture

06-October-2016

* Detection in the presence of noise.

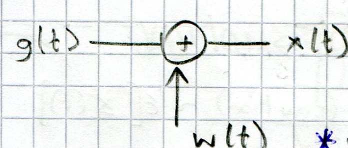
↳ assume this to be AWGN.

↳ always constant



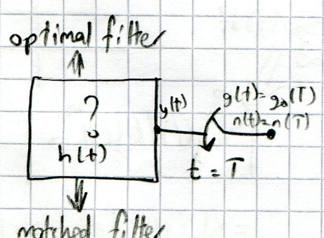
$1 \rightarrow g(t) \quad 0 \le t \le T$

$0 \rightarrow -g(t) \quad 0 \le t \le T$



* white

* PSD $= \frac{N_0}{2}$



$y(t) = x(t) * h(t)$

$y(t) = (g(t) + w(t)) * h(t)$

$= (g(t) * h(t)) + (w(t) * h(t))$

* Optimal filter = Best for that kind, no other filter can go above it.

minimize BER $\longrightarrow$ maximize SNR

$$\eta = \frac{\text{instantenous signal power } t = T}{\text{average noise power}}$$

* find $h(t)$ to make η max!

$$\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}$$

$$g_0(t) = g(t) * h(t)$$
$$= \mathcal{F}^{-1}\{G(f) \cdot H(f)\}$$
$$= \int_{-\infty}^{\infty} G(f) H(f) e^{j2\pi f t} \, df$$

$$g_0(T) = \int_{-\infty}^{\infty} G(f) H(f) e^{j2\pi f T} \, df$$

$$E[n^2(t)] =$$

Input Noise $N_0/2$ $\longrightarrow$ $\boxed{h(t) \quad H(f)}$ $\longrightarrow$ output noise

PSD $= \frac{N_0}{2} |H(f)|^2$

$$S_N(f) = S_w(f) |H(f)|^2$$
$$= \frac{N_0}{2} |H(f)|^2$$

$$\eta = \frac{\left| \int_{-\infty}^{\infty} G(f) H(f) e^{j2\pi f T} \, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df}$$

Schwarz Inequality

$\phi_1(x)$ $\qquad\qquad$ $\phi_2(x)$

$\int_{-\infty}^{\infty} |\phi_1(x)|^2 < \infty$ $\qquad\qquad$ $\int_{-\infty}^{\infty} |\phi_2(x)|^2 < \infty$

$$\left|\int_{-\infty}^{\infty} \phi_1(x)\,\phi_2(x)\,dx\right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx$$

$$\phi_1(x) = k\,\phi_2^*(x)$$

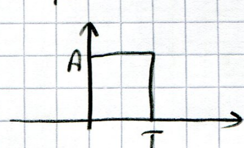
$$\therefore \eta \leq \frac{\int_{-\infty}^{\infty} |G(f)|^2 df \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}{\frac{N_0}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}$$

$$\eta \leq \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df$$

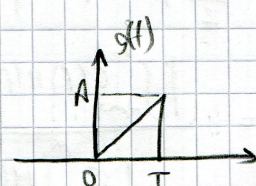
$$\eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \implies \text{if only if } H(f) = K\,G^*(f)\, e^{-j2\pi f T}$$

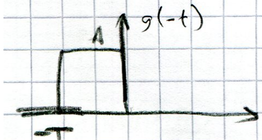
$$h(t) = K\,g(T-t) \longrightarrow MF$$

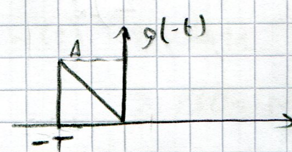
$h_{opt}(t)$

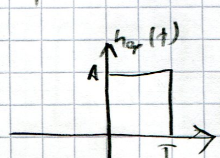


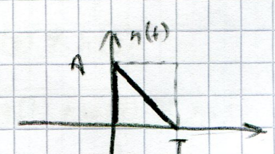
$h_{opt}(t)$??

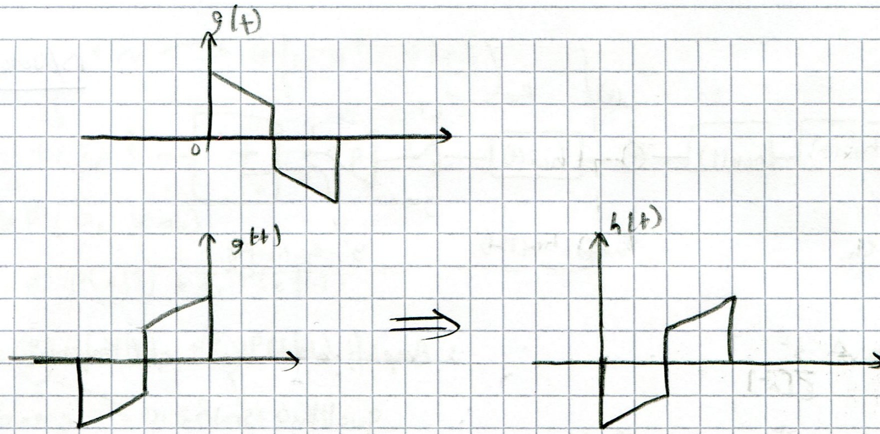






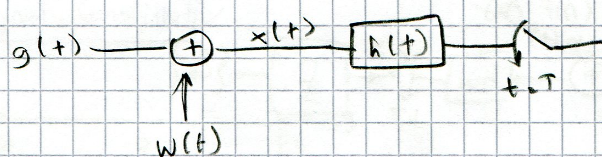






$$\eta_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2 \, df$$

Simple Realization of MF : Correlator Receiver



$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$h(t) = g(T-t)$$

$$h(\tau) = g(t-\tau)$$

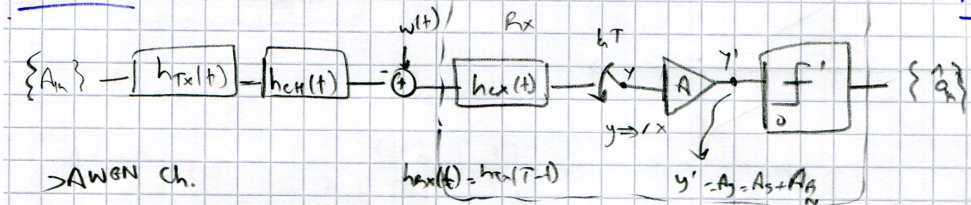
$$h(t-\tau) = g(T-t+\tau)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, g(T-t+\tau)\, d\tau$$

$$y(t=T) = \int_{0}^{T} x(\tau)\, g(\tau)\, d\tau$$

Lecture

13/October/2016



>AWGN ch.

$$SNR_y \triangleq \frac{s^2}{E[n^2]}$$

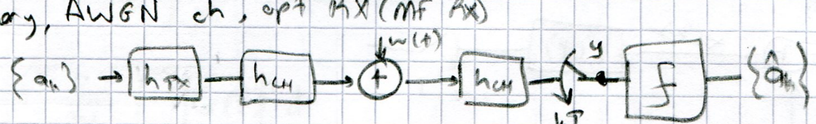
$$SNR_{y'} \triangleq \frac{(As)^2}{E[(An)^2]} = \frac{A^2 s^2}{E[A^2 n^2]}$$

$$= \frac{\cancel{A^2} s^2}{\cancel{A^2} E[n]} = SNR_y$$

∴ Amplification does not improve the quality SNR.

Pe Analysis

- Binary, AWGN ch, opt RX (MF RX)



symbol by symbol det optimal if there is no ISI.

y = Decision variable

$$E[n^2] - \cancel{E^2[n]}^{0}$$

$$y = S + n \longrightarrow G\left(\mu = 0,\ \sigma_n^2 = \frac{E_b N_0}{2}\right)$$

$$f_N(n) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{E_b N_0}{2}}} e^{-n^2/2E_bN_0/2}$$

$$E_b \triangleq \int_{-\infty}^{\infty} h^2_a(t)\,dt$$

$$S = \pm \int h_a^2(t)\,dt$$

$$= \pm E_b$$

$y = s + \underset{\sim}{n} \longrightarrow G\left(0; \sigma^2 = \frac{E_b N_0}{2}\right)$

$\downarrow$

$\pm E_b$

$$P_e \triangleq P(\hat{a}_k \neq a_k)$$

$$= f_1 \, P(e|1) + f_1 \, P(e|-1)$$

$$= f_1 \, P(-1|1) + f_1 \, P(1|-1)$$

* if binary $\quad = 0.5 \qquad = 0.5$

$\overset{Rx}{\ } \ \overset{Tx}{\ }$

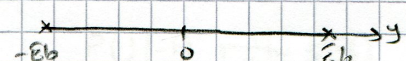
* $P(-1|1)$

1 fixed : $s = E_b$

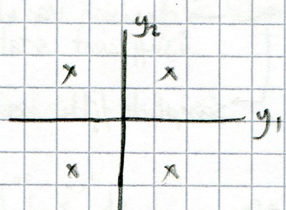
$$y = E_b + \underset{\sim}{n}$$

$$P(-1|1) = P(y < 0|1)$$

Signal Constillation



QPSK



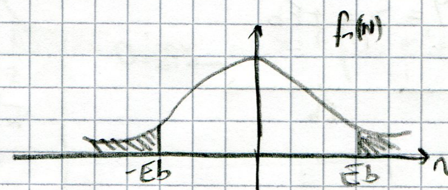
$$= P(\pm E_b + n_N < 0|1)$$

$$= P(E_b + \underset{\sim}{n} < 0)$$

$$= P(\underset{\sim}{n} < -E_b)$$

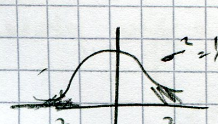
$$= \int_{-\infty}^{-E_b} f_N(n)\, dn$$

PDF



$$= \int_{E_b}^{\infty} f_N(n)\, dn = \int_{E_b}^{\infty} \frac{1}{\sqrt{\pi E_b N_0}} e^{-\frac{n^2}{E_b N_0}} dn$$

$$\operatorname{erfc}(z) \triangleq \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-x^2} dx$$



$$G\left(0; \sigma^2 = \frac{1}{2}\right) \rightarrow 2 \int_{z}^{\infty} \frac{1}{\sqrt{2\pi \frac{1}{2}}} e^{-\frac{x^2}{2 \cdot 1/2}}$$

$$= \int_{Eb}^{\infty} \frac{1}{\sqrt{\pi E_b N_0}} e^{-\frac{n^2}{E_b N_0}} dn$$

$$\frac{n}{\sqrt{E_b N_0}} = x$$

$$= \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi E_b N_0}} e^{-x^2} \sqrt{E_b N_0}\, dx$$

$$= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

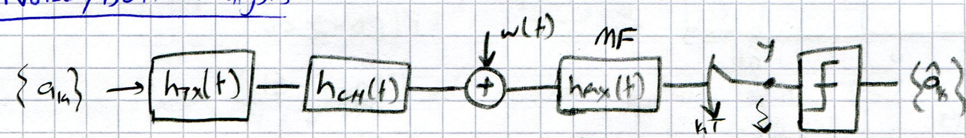
$$P_e = \frac{1}{2} P(1|-1) + \frac{1}{2} P(-1|1)$$

$$= \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

Lecture

18/October/2016

Noise/BER Analysis



$h_{Rx}(t) = h_{Tx}(T-t)$

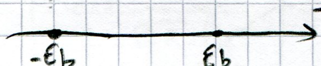
$y = S + N$ → decision variable "sufficient statistic"

N → sampled filter noise

- PAM
- AWGN

→ In the absense of noise ($w(t) = 0$)

$$Y = S = \pm E_b$$



→ $P_e = 0$

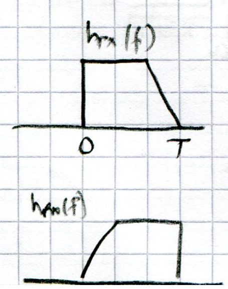
$y = \pm E_b + n$ → $G\left(0, \sigma_N^2 = \frac{N_0 E_b}{2}\right)$

$$\sigma_N^2 \triangleq \frac{N_0}{2} \int_{-\infty}^{\infty} |H_{Rx}(f)|^2 \, df$$

since MF, $|H_{Rx}(f)|^2 = H_{Tx}(f)^2$

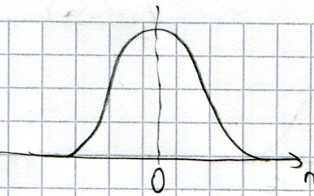
$$\sigma_N^2 = \frac{N_0}{2} \int |H_{Tx}(f)|^2 df = \frac{N_0}{2} \int h_{Tx}^2(t) \cdot t = \frac{N_0 E_b}{2}$$

*



Fourier magnitude of both will be same because their shape is same, only there will be some phase. Just because inverse.

$$f_N(n) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{N_0 E_b}{2}}}\, e^{-\frac{n^2}{2\frac{N_0E_b}{2}}}$$



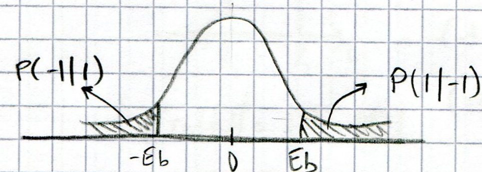
$$P_e \triangleq P(\hat{a}_k \neq a_k)$$

$$= p_1 P(-1|1) + p_1 \underbrace{P(1|-1)}_{P(\hat{a}_k = 1 | a_k = 1)}$$

probability of transmitting a "1"

$$= \frac{1}{2}\Big[\overbrace{P(-1|1)}^{y<0} + \overbrace{P(1|-1)}^{y>0}\Big]$$

$$= \frac{1}{2}\Big[\underbrace{P(E_b + n < 0)}_{P(n < -E_b)} + \underbrace{P(-E_b + n > 0)}_{P(n > E_b)}\Big]$$

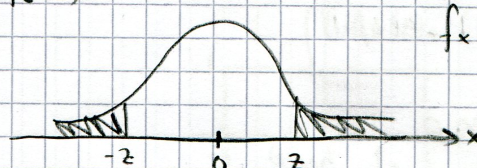


$$P(1|-1) = P(n > E_b)$$

$$\triangleq \int_{E_b}^{\infty} \frac{1}{\sqrt{\pi}\sqrt{E_bN_0}}\, e^{-\frac{n^2}{E_bN_0}}\, dn$$

Defn: $\mathrm{erfc}(z)$

$$= 2\int_z^{\infty} \frac{1}{\sqrt{\pi}} e^{-x^2}\, dx$$



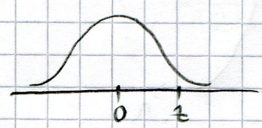
$$f_x(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}\, dx$$

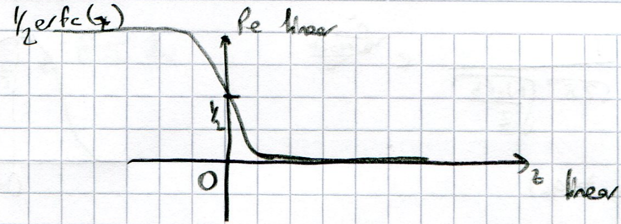
$$x : G\left(0, \sigma^2 = \frac{1}{2}\right)$$

$$\frac{n}{\sqrt{E_bN_0}} = x$$

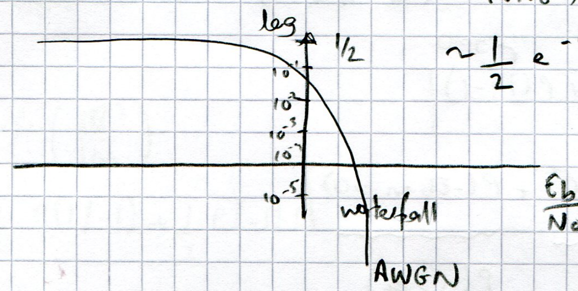
$$P(1|-1) = \frac{1}{\sqrt{\pi}\,\cancel{\sqrt{E_bN_0}}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-x^2}\, dx\, \cancel{\sqrt{E_bN_0}}$$

$$\underbrace{P_e}_{\text{BER}} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$





$\frac{E_b}{N_0} > 0 \qquad P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad$ log - log



$\sim \frac{1}{2} e^{-E_b/N_0}$

$\frac{E_b}{N_0}$ [dB]

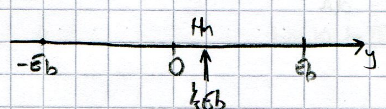
AWGN ch $\rightarrow h_{CH}(t) = \alpha\,\delta(t - t_0)$

factory ch $\rightarrow h_{CH}(t) = \tilde{\alpha}\,\delta(t - t_0)$

Even if $E[\tilde{\alpha}] = \alpha$, still

$P_{e,\text{factory}} \gg P_{e,\text{AWGN}}$

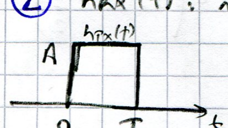
Exercise: 1



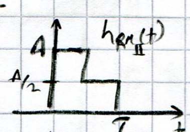
$P_e = ?$

$P_e = \frac{1}{2}\left[P(-1/1) + P(1/-1) \right]$

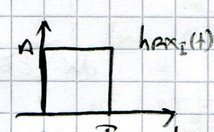
2 $h_{RX}(t)$: not a MF



$\Longrightarrow$



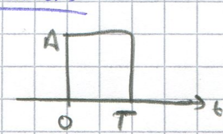
What is the error here $h_{TX}(t) = h_{RX}(t)$.



$P_{e,I} \ll P_{e,II}$

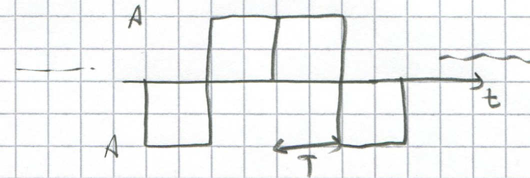
Quiz 2-Answers

$h_{Tx}(t) =$



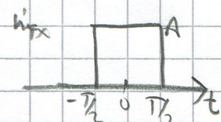
$\dots, -1, 1, 1, -1, \dots$

$x(t) = \sum a_k \, \delta(t - kT - T_0)$

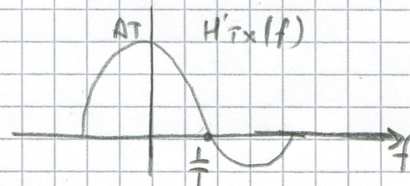


$$S_x(f) = \frac{1}{T} \left| H_{Tx}(f) \right|^2$$

$$h_{Tx}(t) = h'_{Tx}\left(t - \frac{T}{2}\right)$$



$$\left| H'_{Tx}(f) \right| = \left| H_{Tx}(f) \right|$$



$$S_x(f) = \frac{1}{T} A^2 T^2 \, \mathrm{sinc}^2(fT)$$

$$= A^2 T \, \mathrm{sinc}^2(fT)$$