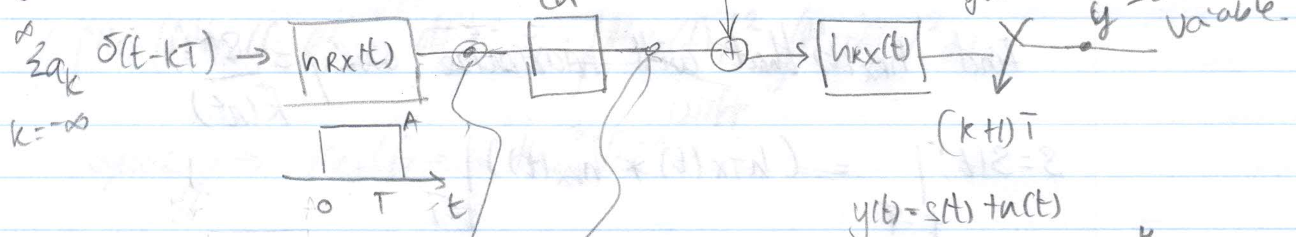
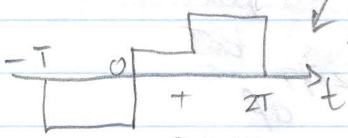
Mary - PAM





a_{-1}	a_0	a_1
-3	1	3
00	10	11

$y(t) = s(t) + n(t)$

wlog, try to detect 0^{th} symbol

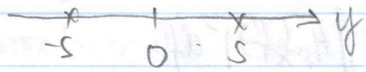
→ focus on $t \in [0,T]$

wlog, Binary PAM

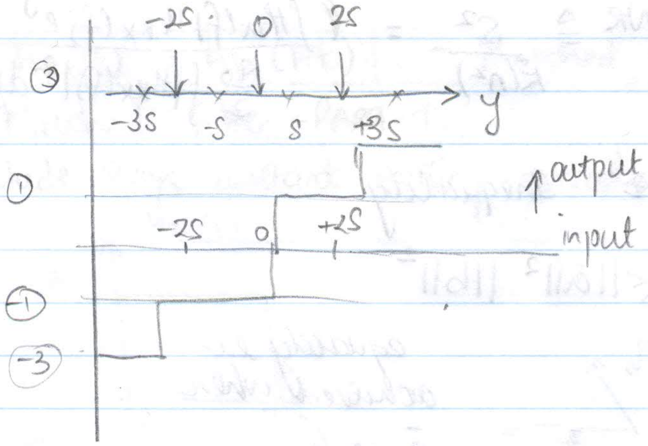
$y(T)$: $y = \pm s + n$ ← $G(0;\sigma)$

↑ signal ↑ noise term

if there were no noise $y = \pm s$



Threshold detector 4-ary PAM



θ For a given Tx filter, $h_{TX}(t)$, find the optimum Rx filter, $h_{RX}(t)$

Find $h_{RX}(t)$ that will maximize $SNR_y = \frac{S^2}{E(n^2)}$

$$S = S(t)\Big|_{t=T} = \left(h_{TX}(t) * h_{RX}(t)\right)\Big|_{t=T}$$

$$= \left(H_{TX}(f)\, H_{RX}(f)\, e^{j2\pi ft}\right)_{t=T}$$

$$= \int H_{TX}(f)\, H_{RX}(f)\, e^{j2\pi fT}\, df$$

$$E(n^2) = \text{Power}(n) = \text{Power}\big(\overbrace{w(t) * h_{RX}(t)}^{n(t)}\big)_{t=T} = \text{Power}(n(t))_{t=T}$$

$$\text{Power}(n(t)) = \int_{-\infty}^{\infty} S_N(f)\, df$$

$$= \int_{-\infty}^{\infty} S_n(f)\, |H_{RX}(f)|^2\, df$$

$w(t) \rightarrow \boxed{h_{RX}(t)} \rightarrow n(t)$

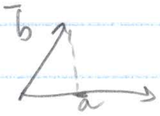
$S_N(f) = \underbrace{S_n(f)}_{\frac{N_0}{2}}\, |H_{RX}(f)|^2$

$$= \frac{N_0}{2} \int |H_{RX}(f)|^2\, df$$

$$SNR \triangleq \frac{S^2}{E(n^2)} = \frac{\left(\int H_{TX}(f)\, H_{RX}(f)\, e^{j2\pi fT}\, df\right)^2}{\frac{N_0}{2}\int |H_{RX}(f)|^2\, df}$$

Schwarz's Inequality

$$\langle \bar{a}, \bar{b} \rangle^2 \le \|\bar{a}\|^2\, \|\bar{b}\|^2$$



equality is achieved when $\bar{a} = \alpha \bar{b}$

$$\langle a(t), b(t) \rangle = \int_{-\infty}^{\infty} \underset{H_{Rx}(f)}{a(t)} \, \underset{H_{Tx}(f)}{b(t)} \, dt$$

$$\left(\int H_{Rx}(f) H_{Tx}(f) e^{j2\pi f T} df \right)^2 \leq \int |H_{Rx}(f)|^2 \int |H_{Tx}(f)|^2 df$$

equality $\rightarrow$ $H_{Rx}(f) = \alpha H_{Tx}^{*}(f) e^{-j2\pi f T}$

$$SNR \leq \frac{\int |H_{Rx}(f)|^2 df \int |H_{Tx}(f)|^2 df}{\frac{N_0}{2} \int |H_{Rx}(f)|^2 df}$$

$$SNR_{max} = \frac{2}{N_0} \int |H_{Tx}(f)|^2 df.$$

Note*.

$$\boxed{\begin{array}{l} g(t) \leftrightarrow G(f) \\ g^*(t) \leftrightarrow G^*(f) \\ g(t-T) \leftrightarrow G(f) e^{-j2\pi f T} \end{array}}$$

$h_{Tx}(T-t) \leftrightarrow ?$

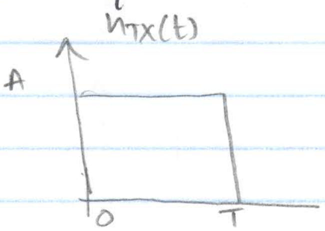
$h_{Tx}(-t) \rightarrow h_{Tx}(-(t-T)) = h_{Tx}(T-t).$

$$\begin{array}{cc} h_{Tx}(-t) & h_{Tx}(-(t-T)) \\ \updownarrow & \updownarrow \\ H_{Tx}^{*}(f) & H_{Tx}^{*}(f) e^{-j2\pi f T} \end{array}$$

$\boxed{h_{Rx}(t) = h_{Tx}(T-t).}$ – Matched filter

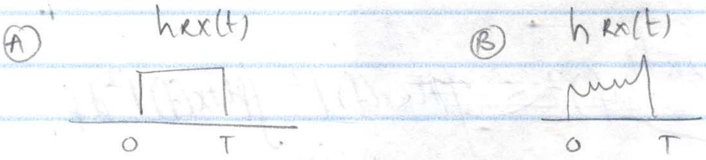
$\rightarrow$ See Notes – Matched filter – PART-1.

The magnitude stays constant during convolution



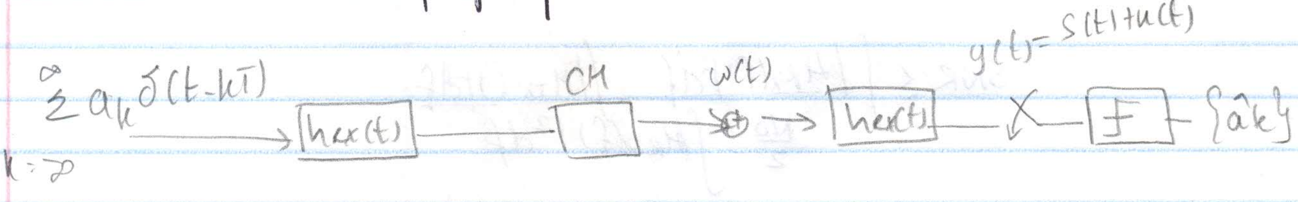
$$\text{Energy} \triangleq \int_{-\infty}^{\infty} h_{Tx}^2(t) \, dt = \int |H_{Tx}(f)|^2 df.$$

SNR is related to the energy of the pulse.



Binary - PAM

They are both good since they have the same energy. & the receivers are matched properly



$P_e \triangleq P(\hat{a}_k \neq a_k)$.

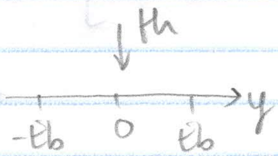
Special case: Binary PAM

$a_k = \pm 1$

$$P_e = P(\hat{a} \neq a)$$
$$= P_1 \cdot P(\hat{a} \neq a \mid a=1) + P_{-1} \cdot P(\hat{a} \neq a \mid a=-1)$$
$$= \frac{1}{2} P(\hat{a} = -1 \mid a=1) + \frac{1}{2} P(\hat{a} = 1 \mid a=-1)$$
$$= \frac{1}{2} P(\hat{a} = -1 \mid y = E_b + n) + \frac{1}{2} \hat{P}(\hat{a} = 1 \mid y = -E_b + n)$$

$$y = \begin{cases} s+n & a=1 \\ -s+n & a=-1 \end{cases}$$

$$\text{MF}: \quad y = \begin{cases} E_b + n, & a=1 \\ -E_b + n, & a=-1 \end{cases}$$



$P(\hat{a} = -1 \mid y = E_b + n)$.

$P(\hat{a}=-1 \mid y=E_b+n)$

$= P(y<0 \mid y = E_b+n)$

$= P(E_b+n<0)$

$= P(n<-E_b)$

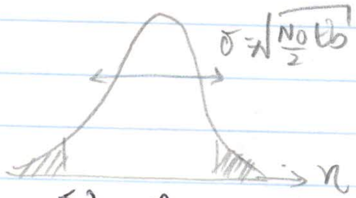
$n : G(0;\sigma)$

$E(n^2) = \int S_N(f)df = \int S_w(f)|H_{RX}(f)|^2 df$

$= \frac{N_0}{2}\int |H_{RX}(f)|^2 df$

$= \frac{N_0}{2}\int |H_{RX}(t)|^2 dt = \frac{N_0}{2} E_b.$

PDF of noise (Gaussian).



$P(n<-E_b) = P(n>E_b) = \int_{\infty}^{\infty} f_N(n)dn = \int_{E_b}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{n^2}{2\sigma^2}} dn$