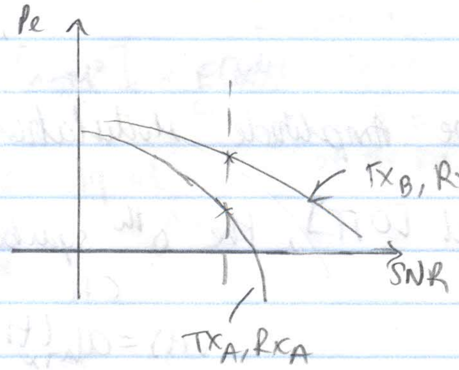
SYSC 4600 - lecture 7 26/09/2014

* what is the structure of the optimal TX and RX?
* what is the performance of a TX, RX pair, not necessarily optimized (for any given design)?

Def performance: Pe: as a function of SNR



what should I be worried about? (Digital or analog system).

- Noise
- Interference
- Channel (Medium of communication).

} Study the impact one at a time.

Setting:

noise ✓ AWGN

(assume none) interference x

channel (perfect).

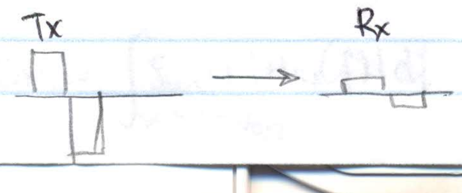
Ideal Channel : $h_{CH}(t) = \alpha\delta(t-t_p)$. always be attenuation x delay

wlog, $\alpha = 1$, $t_p = 0 \rightarrow h_{CH}(t) = \delta(t)$: ~~short~~ short circuit channel

Example of a trivial case would be

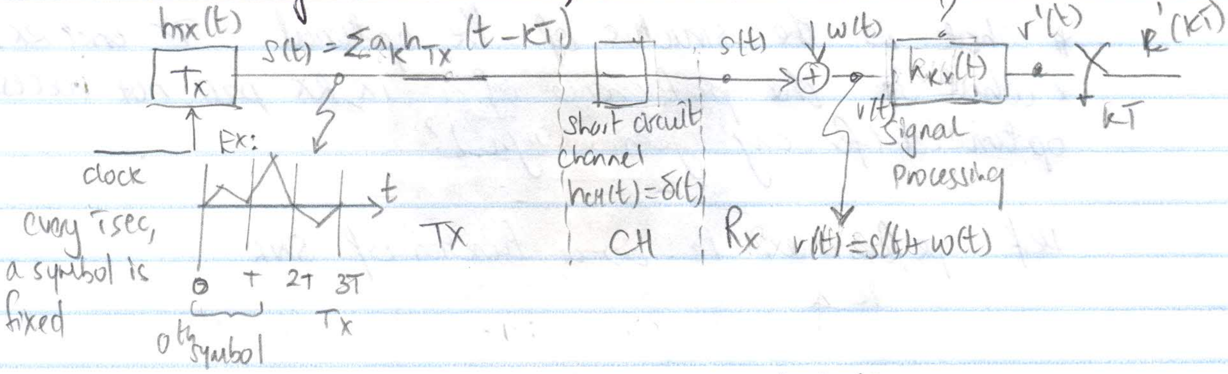
noise x
interference x
$h_{CH}(t) = \alpha\delta(t-t_p)$

Assume $\alpha << 1$ Pe=0. (not performing).



(not necessarily optimum)

Q) for a given transmitter, obtain the best receiver?



Meaning of M-ary PAM - Pulse Amplitude Modulation

wlog, consider the interval $[0,T]$, the 0^{th} symbol

Tx: $s(t) = a h_{TX}(t)$

CH: $v(t) = a h_{TX}(t) + w(t)$

Recent signal + noise after processing

$v'(t) = (s(t) + w(t)) * h_{Rx}(t)$

$= \underbrace{(s(t) * h_{Rx}(t))}_{s'(t)} + \underbrace{(w(t) * h_{Rx}(t))}_{n(t)}$. white noise after processing, filtered white noise, not white anymore.

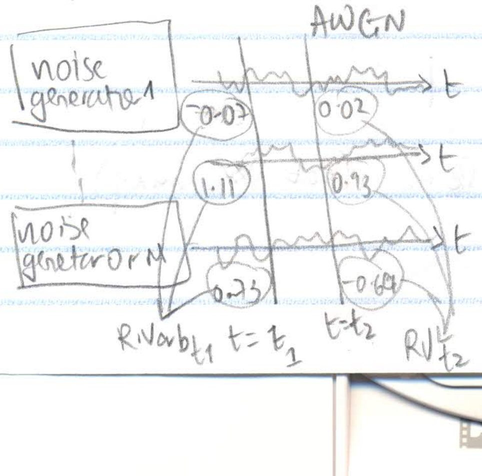
$r' : r'(t=T) = v'(T)$

$= s'(T) + n(T)$

$= \underbrace{s'}_{} + n$

RV due to the fact that a is Random Variable → M different values, equally-likely

M=4



AWGN → RV_{t_1}, RV_{t_2} : Gaussian RV

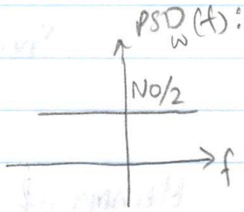
$x : G(m, \sigma)$

↓

Mean=0

$$f_X = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Relation → to PSD

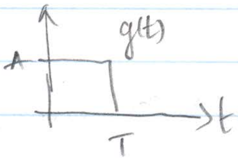


white noise : ∞ power

$$\sigma^2 = E[X-\mu^2] = \underbrace{E(x^2)}_{\text{average total power of the RV}}$$

→ instanteneous power

Deterministic signal



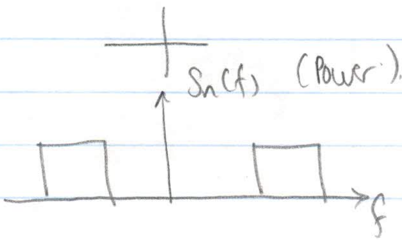
$$\text{Energy of } g(t) \triangleq \int_{-\infty}^{\infty} g^2(t)\,dt = A^2T \text{ joules}$$

Power of g(t) = A^2 watts.

$P \times T =$ Energy

$W \times s = j$

filtered noise in the signal BW



$\underbrace{\text{random process}}$

$w(t) \longrightarrow \boxed{h(f)} \longrightarrow n(t)$

$S_w(f)$ $\qquad S_n(f) = S_w(f)\,|H(f)|^2$

$E[n^2] = \sigma^2$ $\qquad \underset{G(0;\infty)}{w(t)} \longrightarrow \boxed{h_{Rx}(t)} \longrightarrow n(t)$

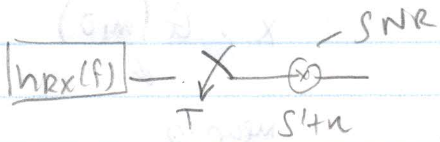
$n : G(0, \sigma)$

$$\therefore \text{Total Power} = \int_{-\infty}^{\infty} S_n(f)\,df$$

$$\sigma^2 = E[n^2] = \int_{-\infty}^{\infty} \underbrace{S_w(f)}_{No/2} |H_{Rx}(f)|^2\,df$$

$$\sigma^2 = \frac{N_0}{2} \int |H_{Rx}(f)|^2 df$$

$$SNR = \frac{S'^2}{E[n^2]}$$



Meaning of MF = $h_{Rx}(t) = h_{Tx}(T-t)$ — Matched filter