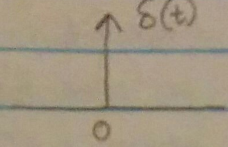
09/09/15

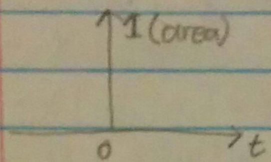
Lecture 3

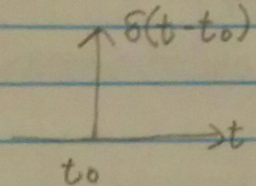
Delta Function

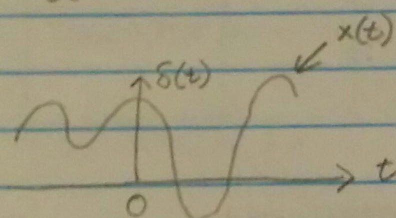


$\delta(0)$: undefined

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt = 1$$





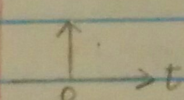


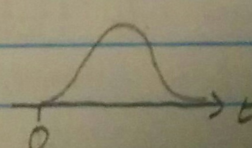
$$x(t)\delta(t) = \overbrace{x(0)}^{\text{constant}}\delta(t)$$

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

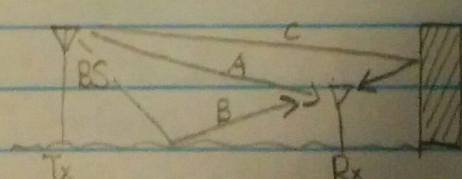
Impulse Response

$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t)$: impulse response

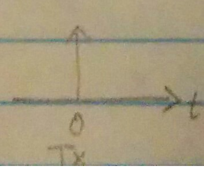


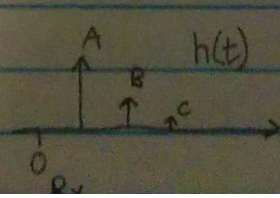


Ex: Wireless Multipath Channel



(Not to scale)



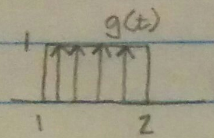


$h(t)$ $\overset{\text{in the limit}}{\Rightarrow}$

General Case:

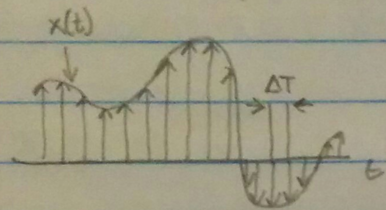
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

Ex:



$$\tilde{g}(t) = \left[\delta(t-1.125) + \delta(t-1.375) + \delta(t-1.625) + \delta(t-1.875)\right]\frac{1}{4}$$

$$\text{Area}(g(t)) = 1 \qquad \text{Area}(\tilde{g}(t)) = 4$$



$$\sum_{n=-\infty}^{\infty} x(n\Delta T)\delta(t-n\Delta T)\Delta T$$

$$\hookrightarrow x(t) = \lim_{\Delta T \to 0} \sum_{-\infty}^{\infty} x(n\Delta T)\delta(t-n\Delta T)\Delta T$$

$$x(t) = \lim_{\Delta T \to 0} \sum_{-\infty}^{\infty} x(n\Delta T)\delta(t-n\Delta T)\Delta T \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(n\Delta T)\delta(t-n\Delta T) \longrightarrow \boxed{h(t)} \longrightarrow x(n\Delta T)h(t-n\Delta T)$$

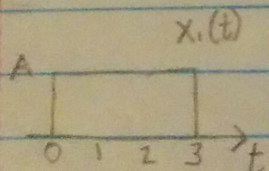
$$\therefore y(t) = \lim_{\Delta T \to 0} \sum_{-\infty}^{\infty} x(\overbrace{n\Delta T}^{\tau})h(t-n\Delta T)\Delta T$$

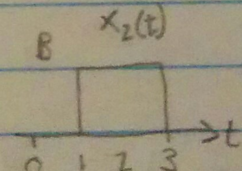
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \quad \text{: convolution}$$

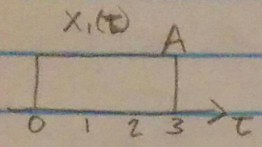
Ex:

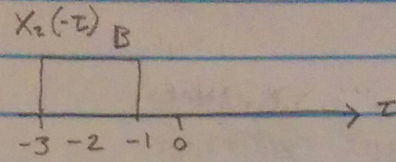
$$x_1(t) * x_2(t) = y(t)$$

$$\int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)\,d\tau = y(t)$$



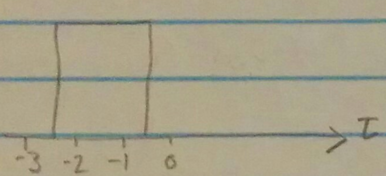


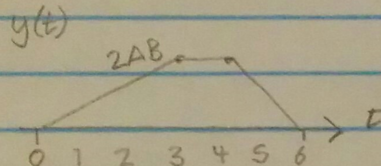




$$x_2(-[\tau - t]) = x_2(t-\tau)$$

$t = 0.5$





Observe: $\text{length}[y(t)] = \text{length}[x_1(t)] + \text{length}[x_2(t)]$

Convolution results in broadening of the transmitted signal

Will result in self-interference (ISI)

Exception: $\underbrace{h(t) = \alpha\delta(t-t_0)}_{\text{ideal channel}}$

$$y(t) = x(t) * [\alpha\delta(t-t_0)]$$
$$\triangleq \underbrace{\int \alpha\,\delta(\tau-t_0)\,x(t-\tau)\,d\tau}$$

$$= \int \alpha\,\underbrace{x(t-t_0)}_{\text{scaling}}\,\delta(\tau-t_0)\,d\tau$$
$$= \alpha x(t-t_0)\int \delta(\tau-t_0)\,d\tau$$
$$= \alpha x(t-t_0)$$

$x(t) * \delta(t) = x(t)$ $\quad\therefore$ Delta function is the identity function with respect to convolution.

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$$

$X(f)$ $\quad \underset{H(f)}{LTI} \quad$ $Y(f) = X(f)H(f)$

$$\mathcal{F}\{x(t) * h(t)\} \neq X(f) * H(f)$$
$$= X(f)H(f)$$

If $x(t)$: pure sinusoidal tone, $y(t)$ is also a sinusoidal tone.

→ sinusoidals are eigenfunctions of LTI systems

$$x(t) = e^{j2\pi f_c t}$$

$$e^{j2\pi f_c t} \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) \triangleq \int_{-\infty}^{\infty} h(\tau) e^{j2\pi f_c (t-\tau)} d\tau$$
$$= e^{j2\pi f_c t} \left(\int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_c \tau} d\tau \right) = H(f_c)$$

$$= H(f_c) e^{j2\pi f_c t}$$

$H(f_c)$ = complex constant = $\alpha e^{j\theta}$

$$= \underset{\text{scaling}}{\alpha} e^{j(2\pi f_c t + \overset{\text{phase}}{\theta})}$$

periodic, T_c

$$x(t) = \sum \alpha_i \cos(2\pi n f_0 t + \theta_c)$$

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Lecture 4

$$x(t) \longrightarrow \underset{}{\overset{\text{LTI}}{\boxed{h(t)}}} \longrightarrow y(t) = x(t) * h(t)$$

$e^{j2\pi f_0 t}$

$$y(t) \triangleq e^{j2\pi f_0 t} * h(t)$$
$$= \int h(\tau) e^{j2\pi f_0 (t-\tau)} d\tau \quad \text{particular tone, } f = f_0$$
$$= e^{j2\pi f_0 t} \underbrace{\int h(\tau) e^{-j2\pi f_0 \tau} d\tau}_{H(f)|_{f=f_0}}$$

$H(f)|_{f=f_0}$: complex constant

$$y(t) = H(f_0) e^{j2\pi f_0 t}$$

$e^{j2\pi f_0 t}$: exception, shape is preserved

special case: periodic $x(t)$, $T_0 = 1/f_0$

complex constants

$$x(t) = \underbrace{\sum_{-\infty}^{\infty} a_n e^{j2\pi n f_0 t}}_{\text{countable - infinitely many}} \quad : \text{FS}$$

generic case: $x(t)$: not periodic

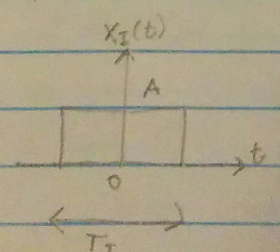
$$x(t) = \underbrace{\int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df}_{\text{uncountable - infinitely many}} \quad : \text{FT}$$

$$x(t) = \int_f X(f) e^{j2\pi f t} df \rightarrow \overset{\text{LTI}}{\boxed{h(t)}} \rightarrow y(t) = \int \underbrace{X(f)H(f)}_{Y(f)} e^{j2\pi f t} df$$

$$\frac{x(t)}{X(f)} \rightarrow \boxed{\frac{h(t)}{H(f)}} \rightarrow \begin{matrix} y(t) = x(t) * h(t) \\ Y(f) = X(f) * H(f) \end{matrix}$$

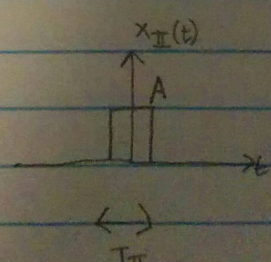
Bandwidth

I



$P_I = A^2$

II



$P_{II} = A^2$

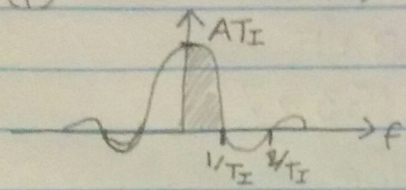
$T_I = 10 T_{II}$

$R = 1/T \quad \therefore R_{II} = 10 R_I$

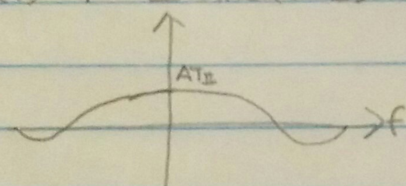
$E_I = A^2 T_I \quad E_{II} = A^2 T_{II} \quad E_I = 10 E_{II}$

Reliability vs. Rate (often conflicting)

$X_I(f) = AT_I \,\text{sinc}(fT_I)$



$X_{II}(f) = AT_{II} \,\text{sinc}(fT_{II})$



$BW_{II} = 10 BW_I \qquad \therefore$ More R $\rightarrow$ More BW

*BW: always read from +ive frequencies

* Various definition's for BW

- absolute BW [finite BW signals]
- null BW
- 95% BW

1024 ary $\rightarrow$ 10 bits/symbol

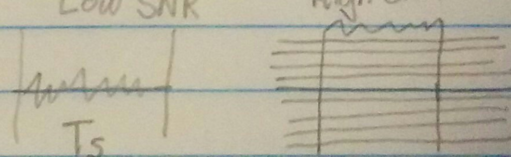
$R\uparrow \rightarrow$ BW$\uparrow$

$R\uparrow \rightarrow$ SNR$\uparrow$

$R\uparrow \rightarrow$ Antenna$\uparrow$

Binary | Mary

Low SNR | High SNR



$X(0) = \int x(t)\,dt$ ↔ Dual

$x(0) = \int X(f)\,df$

$SNR = \dfrac{P_{signal}}{P_{noise}\big|_{\text{in signal BW}}}$

Ex:

$N_0 = -174\ dBm/Hz$

$B = 200\ kHz$

$F = 7\ dB$

Linear

$P = N_0 * B \times F$

$= 10^{-174/10} \times 2\times10^5 \times 10^{7/10}$

$=$ mWatts

Log

$P = N_0 + B + F$

$= -174 + 53 + 7$

$= -114\ dBm$

$\overbrace{R\uparrow \Rightarrow B\uparrow}^{\text{Linear}} \Rightarrow P_{noise}\uparrow \Rightarrow \underbrace{SNR\downarrow \Rightarrow \text{bits/sym}\downarrow}_{\text{log}} \Rightarrow R\downarrow$