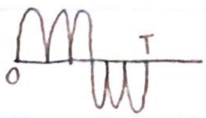
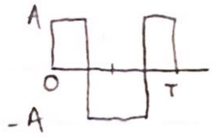
Dec 4, 2015

Ex $S_0(t)$



$E = E_0 = \int_0^T S_0(t)^2 \, dt$

$S_1(t)$



$E_1 = A^2 T$

Question: What is probability of error?

$P_e = P_0 P_{e|0} + P_1 P_{e|1}$ Binary $P_0 = P_1 = 1/2$

$P_e = \frac{1}{2}(P_{e|0} + P_{e|1})$

Binary case $P_{e|0} = P_{e|1}$ $\Rightarrow$ $\therefore P_e = \underbrace{P_{e|0}}_{P_{e,0\to 1}} = \underbrace{P_{e|1}}_{P_{e,1\to 0}}$

We know P_e is a function of the distance in constellation

$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\left(\frac{d_{01}}{2\sqrt{N_0}}\right)}$$

we need distance

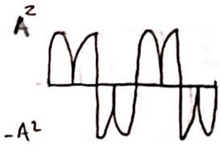
We need to get the signal space and calculate the distance

① We need to first calculate basis functions

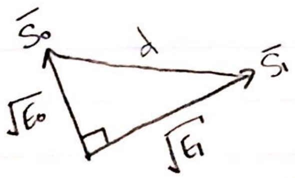
If $(S_0(t), S_1(t)) = 0 \rightarrow$ two signals are orthogonal $\rightarrow \int_0^T S_1(t) S_0(t)\, dt$

By inspection

$S_1(t)\ S_0(t)$



$\int_0^T \text{fxn} = 0$



$d^2 = E_1 + E_0 = 2E$

$E_{av} = E = \frac{E_0 + E_1}{2}$

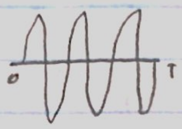
$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{2E}{4N_0}} = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E}{2N_0}}}$$

$\therefore$ 3dB worse than antipodal

Bandpass Modulation

$$\left.\begin{aligned} s_1(t) &= A\cos(2\pi f_c t + \alpha) \\ s_2(t) &= A\sin(2\pi f_c t + \beta) \end{aligned}\right\} \quad 0 \le t \le T \qquad \frac{T}{T_c} = n = Tf_c$$

$$E = \frac{A^2 T}{2} \leftarrow$$



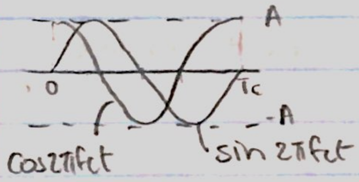
$$E_2 = E_1$$

$$A = \sqrt{E}\sqrt{\frac{2}{T}} \Rightarrow s(t) = \sqrt{E}\sqrt{\frac{T}{2}}\cos(2\pi f_c t + \alpha)$$

$$\boxed{\text{Energy of } (s(t)) = E}$$

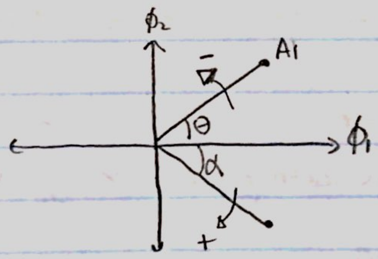
normalized unit energy

we know $\sin \perp \cos \rightarrow \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \perp \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$



$$A\sin 2\pi f_c t = A\cos(2\pi f_c t - \pi/2)$$
$$A\cos 2\pi f_c t = A\sin(2\pi f_c t + \pi/2$$

$$\phi_1 = \sqrt{\tfrac{2}{T}}\cos(2\pi f_c t) \qquad \phi_2 = \sqrt{\tfrac{2}{T}}\sin(2\pi f_c t)$$



$$A_1 = \sqrt{\tfrac{2}{T}}\cos(2\pi f_c t - \theta)$$
$$A_2 = \sqrt{\tfrac{2}{T}}\cos(2\pi f_c t + \alpha)$$

$A_i \cos(2\pi f_c t + \underbrace{\theta}_{\theta = 0})$ ASK

$\underbrace{A}_{= \sqrt{E}\sqrt{\frac{2}{T}}} \cos(2\pi f_c t + \theta_i)$ FSK

$\sqrt{E_i}\sqrt{\frac{2}{T}}\cos(2\pi f_c t + \theta_i)$ QAM

$A\cos(2\pi f_i t)$ PSK

Ex **8PSK**

$$S_i(t) = \sqrt{E}\sqrt{\frac{2}{T}}\cos(2\pi f_c t + \theta_i)$$

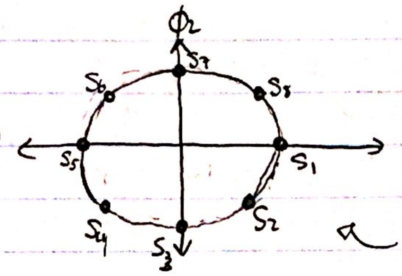
$$= \sqrt{E}\sqrt{\frac{2}{T}}\cos\theta_i \cos(2\pi f_c t) - \sqrt{E}\sqrt{\frac{2}{T}}\sin\theta_i \sin(2\pi f_c t)$$

$$= S_{i1}\ \phi_1(t) + S_{i2}\ \phi_1(t)$$

$$\therefore\ S_{i1} = \sqrt{E}\cos\theta_i \qquad S_{i2} = -\sqrt{E}\sin\theta_i$$

$$E = S_{i1}^2 + S_{i2}^2 = E(\sin^2\theta_i + \cos^2\theta_i) = E$$

signal constellation



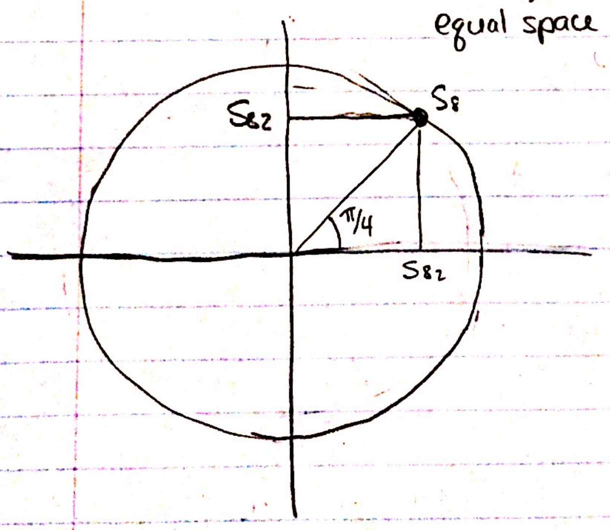
all amplitudes are same

sparser (less probability of error vs )

$$\text{Pe} = \sum_1^8 P_k \text{Pe}_{1k}$$

$$P_k = \frac{1}{8} \Rightarrow \boxed{\text{Pe} = \frac{1}{8}}$$

equal space = equal error



$$S_{81} = \sqrt{E}\cos\left(\frac{\pi}{4}\right) = \sqrt{\frac{E}{2}}$$

$$S_{82} = \sqrt{E}\sin\left(-\frac{\pi}{4}\right) = \sqrt{\frac{E}{2}}$$

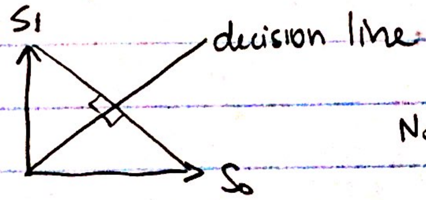
Binary signalling notes (online)

$$(r_1, s_0) - (r_1, s_1) = \int r_1(t)\, s_0(t)\, dt - \int r_1(t)\, s_1(t)\, dt$$

$$= \int r_1(t) \left[s_0(t) - s_1(t) \right] dt$$

$$= \left(r_1, [s_0 - s_1] \right)$$

Decision boundary $(r_1, s_0 - s_1) = 0$



Not to scale