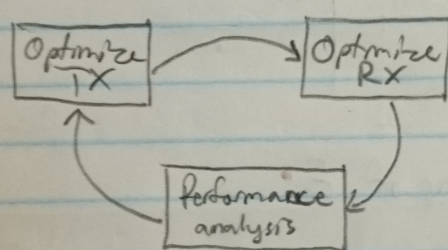
Lecture 22 Nov. 30, 2015

Goal: for a given signaling system, $\{m_i\}_{i=1}^{M} \rightsquigarrow \{s_i(t)\}_{i=1}^{M}$

* Obtain the optimal decision rule (detection strategy)
* Build the receiver
* Performance error analysis (Performance analysis)
* By investigating Performance, Optimize the signaling system



Signal Space Analysis

inner product: $(a(t), b(t)) \triangleq \int_{-\infty}^{\infty} a(t)\, b(t)\, dt$

dot product: $\bar{a} \cdot \bar{b} \triangleq [a_1 a_2 \ldots a_N] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$

$a(t) \equiv \bar{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$

$\bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$

$\bar{a} \cdot \bar{b} = \sum_{j=1}^{N} a_j b_j$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{\sqrt{\bar{a} \cdot \bar{a}}\sqrt{\bar{b} \cdot \bar{b}}}$$

Pair wise Error Probability

$a: l$

$\hat{a}: k$

$$\boxed{P_{e,l\to k} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d_{l,k}}{2\sqrt{N_0}}\right)}$$ ← Should be on cheat sheet

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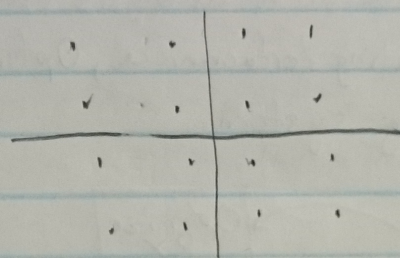
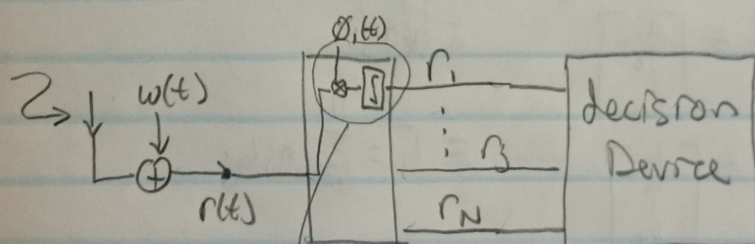
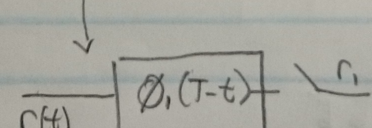


Write d's in terms of E

$$E_i = \int s_i^2(t)\,dt$$
$$= \|s_i\| = \sqrt{s_i \cdot s_i}$$

$$E_{b,av} = \frac{1}{M}\sum_{i=1}^{M} E_i$$





$r_j = s_{ij} + w_j \to \int w(t)\phi_j(t)\,dt \qquad w_j: G(0, \sigma_w^2)$

$\quad \hookrightarrow \int s_i(t)\phi_j(t)\,dt$

$w(t) \to \boxed{\phi_j(T-t)} \to w_j$

$$f_{W_j}(w_j) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{w_j^2}{2\frac{N_0}{2}}}$$

$$f_R(\bar{r} \mid m_i) = \prod_{j=1}^{N} \underbrace{f_{R_j}(\bar{r}_j \mid m_i)}_{\frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(r_j - s_{ij})^2}{2\frac{N_0}{2}}}}$$

$$f_R(\bar{r} \mid m_i) = \frac{1}{(\pi N_0)^{N/2}} e^{-\frac{\sum_{j=1}^{N}(r_j - s_{ij})^2}{N_0}}$$

Likelihood Function

$L(m_i) = f_{\bar{R}}(\bar{r} \mid m_i)$

LLR: log-likelihood function $l(m_i) = \log L(m_i) = -\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{ij})^2$

Optimum decision Rule:

$$\boxed{\begin{array}{c} \text{Set } \hat{m} = m_i \text{ if} \\ P(m_i \text{ sent} \mid \bar{r}) \geq P(m_k \text{ sent} \mid \bar{r}) \end{array}}$$

Maximum a posteriori probability (MAP)

if symbols are equally likely

$$\boxed{\begin{array}{l} \text{set } \hat{m} = m_i \text{ if} \\ f_{\bar{R}}(\bar{r} \mid m_k) \text{ is maximum for } k = i \\ l(m_k) = -\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{ki})^2 \text{ is maximum for } k = i \end{array}}$$

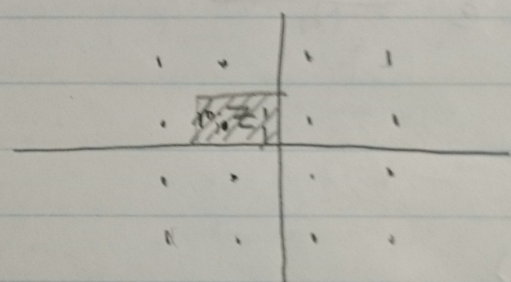
Maximum Likelihood (ML) Rule

$\boxed{\text{Set } \hat{m} = m_i \text{ if}}$

Set $\hat{m} = m_i$ if
$\sum_{j=1}^{N} (r_j - s_{kj})^2$ is minimum for $k = i$
$\leftarrow \|r - s_k\|^2$

Set $\hat{m} = m_i$ if
$\bar{r}$ is in Z_i

Minimum Distance Rule (MD)



Set $\hat{m} = m_i$ if
$\sum_{j=1}^{N} r_j s_{kj} - \frac{1}{2} E_k$ is maximum for $k = i$

Correlator receiver