Lecture 22
Nov 30, 2015

Goal: For a given signaling system, \( E[\sum_{i=1}^{M} s_i^g(t)] = \sum_{i=1}^{M} E[s_i(t)] s_i^g(t) \).

* Obtain the optimal decision rule (detection strategy).
* Build the receiver.
* Performance error analysis (Performance analysis)
* By investigating performance, optimize the signaling system

Diagram:

- Optimize Tx
- Optimize Rx
- Performance analysis

Signal Space Analysis

Inner product: \( (a(t), b(t)) = \int_{-\infty}^{\infty} a(t) b(t) dt \)

Dot product: \( \bar{a} \cdot \bar{b} = [a_0, a_N] [\bar{b}_0, \bar{b}_N]^T \)

\( a(t) = \bar{a} = \begin{bmatrix} a_0 \\ \vdots \\ a_N \end{bmatrix} \)

\( \bar{b} = \begin{bmatrix} \bar{b}_0 \\ \vdots \\ \bar{b}_N \end{bmatrix} \)

\( \bar{a} \cdot \bar{b} = \sum_{j=1}^{N} a_j b_j \)

\( \cos \Theta = \frac{\bar{a} \cdot \bar{b}}{\sqrt{\bar{a} \cdot \bar{a}} \sqrt{\bar{b} \cdot \bar{b}}} \)
Pair WRE Error Probability
\[ a_i: l \]
\[ a_i: k \]
\[ P_{e_i|e_{-i}} = \frac{1}{2} \text{erfc} \left( \frac{d_{x_i}}{2 \sqrt{N_0}} \right) \] ← Should be on cheat sheet

16 QAM

Write \( d_i \)’s in terms of \( E \)
\[ E_i = \int s_i^2(t) \, dt \]
\[ = 11 s_i^\dagger s_i = \sqrt{s_i^\dagger s_i} \]
\[ E_{b,\text{av}} = \frac{1}{M} \sum_{i=1}^{M} E_i \]

\[ Z \xrightarrow{w(t)} r(t) \xrightarrow{r_i(t)} \text{Decision Device} \]

\[ r_i(t-t_0) \]

\[ r(t) \xrightarrow{\Phi_i(t-t_0)} \]

\[ r_3 = s_3 + w_3 \]
\[ \xrightarrow{\int s_i(t) \Phi_j(t) \, dt} w_3 \sim \mathcal{N}(0, \sigma_w^2) \]
\[f_{w_3}(w_3) = \frac{1}{\sqrt{2\pi} N_0^{\frac{1}{2}}} e^{-\frac{w_3^2}{2 N_0}}\]

\[f_R(\bar{r} \mid m_i) = \frac{1}{N} \prod_{j=1}^{N} f_{w_j}(\bar{r}_j \mid m_i)\]

\[= \frac{1}{\sqrt{2\pi N_0^{\frac{1}{2}}}} e^{-\frac{1}{2 N_0} \sum_{j=1}^{N} (r_j - \bar{s}_j)^2}\]

\[f_R(\bar{r} \mid m_i) = \frac{1}{\sqrt{2\pi N_0^{\frac{1}{2}}}} e^{-\frac{1}{2 N_0} \sum_{j=1}^{N} (r_j - \bar{s}_j)^2}\]

Likelihood Function:

\[L(m_i) = f_R(\bar{r} \mid m_i)\]

\[LLR: \text{log-likelihood function} \quad l(m_i) = \log L(m_i) = -\frac{1}{N_0} \sum_{j=1}^{N} (r_j - \bar{s}_j)^2\]

Optimum decision rule:

- Set \( \hat{m} = m_i \) if \( P(m_i \mid \text{sent}(\bar{r})) \geq P(m_k \mid \text{sent}(\bar{r})) \) and symbols are equally likely

<table>
<thead>
<tr>
<th>Maximum a posteriori probability (MAP)</th>
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</thead>
<tbody>
<tr>
<td>Set ( \hat{m} = m_i ) if ( f_R(\bar{r} \mid m_k) ) is maximum for ( k \neq i )</td>
</tr>
<tr>
<td>Maximum Likelihood CML Rule</td>
</tr>
<tr>
<td>( l(m_k) = -\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{ki})^2 ) is maximum for ( k = i )</td>
</tr>
</tbody>
</table>
Set $\hat{m} = m_j$ if $\frac{1}{N} \sum_{j=1}^{N} (r_j - S_k)_j^2$ is minimum for $k = i$.

Set $\hat{m} = m_j$ if $\parallel r - S_k \parallel^2$.

$r_i$ is in $Z_j$.

Minimum Distance Rule (MD)

Correlator receiver