

$$\underline{u}_i = \frac{\underline{w}_i}{\|\underline{w}_i\|} \quad \underline{w}_i = \underline{a}_i - \sum_{j=1}^{i-1} (\underline{a}_i, \underline{u}_j) \underline{u}_j$$

Lecture 19

Nov. 18 2015

$$\underline{a}, \|\underline{a}\|, \underline{a} \cdot \underline{b} = \|\underline{a}\| \|\underline{b}\| \cos \theta$$

$$\theta = \arccos \left(\frac{\underline{a} \cdot \underline{b}}{\|\underline{a}\| \|\underline{b}\|} \right)$$

$$\underline{a} \cdot \underline{b} \rightarrow 0 \Rightarrow \underline{a} \perp \underline{b}$$

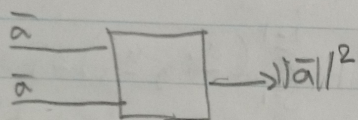
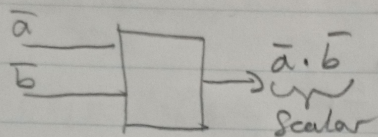
$$\rightarrow \text{max} \Rightarrow \underline{a} = c \underline{b}$$

dot product shows how much the vectors look alike

$$\underline{a} \cdot \underline{a} = \|\underline{a}\|^2 \Rightarrow \|\underline{a}\| = \sqrt{\underline{a} \cdot \underline{a}}$$

Statistics Vectors Waveforms (Signals)
 Correlation = dot product = inner product

dot product Operator



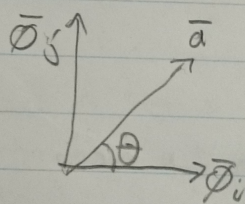
Orthogonal basis Vectors

$$\{\vec{\phi}_i\}_{i=1}^N, \vec{\phi}_i \cdot \vec{\phi}_j = 0, i \neq j$$

$$\vec{\phi}_i \cdot \vec{\phi}_i = 1, \forall i$$

$\{\vec{\phi}_i\}_{i=1}^N$ spans the N -dim space

↳ not unique



decomposition

$$\vec{a} = \sum_{i=1}^N a_i \vec{\phi}_i$$

projection of \vec{a} on the $\vec{\phi}_i$ axis

$$a_i = \vec{a} \cdot \vec{\phi}_i$$

$$= \|\vec{a}\| \|\vec{\phi}_i\| \cos \theta$$

Inner product

$$\int s_1(t) s_2(t) dt$$

$$\int S_1(f) S_2(f) df$$

Orthogonal basis function $s_i: \{\phi_i(t)\}_{i=1}^N \rightarrow \int \phi_i(t) \phi_j(t) dt = \int_{i,j} \begin{cases} 1, i=j \\ 0, i \neq j \end{cases}$