Lecture 19

11/18/15

Vector Space (Vector Algebra)

$\bar{a}$ $\quad \|\bar{a}\|$ $\quad \bar{a}\cdot\bar{b} = \|\bar{a}\|\|\bar{b}\|\cos\theta \qquad \theta = \cos^{-1}\left(\frac{\bar{a}\cdot\bar{b}}{\|\bar{a}\|\|\bar{b}\|}\right)$

$\bar{a}\cdot\bar{b} \to 0 \Rightarrow \bar{a}\perp\bar{b}$
$\qquad\ \searrow \max \Rightarrow \bar{a} = c\bar{b}$
} dot product shows how much the two vectors look alike

Statistics — Vectors — Waveforms

Correlation $\equiv$ dot product $\equiv$ inner product

$\bar{a}\cdot\bar{a} = \|\bar{a}\|^2$

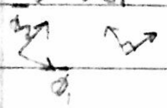
$\Rightarrow \|\bar{a}\| = \sqrt{\bar{a}\cdot\bar{a}}$

$\bar{a}, \bar{b} \to \square \to \bar{a}\cdot\bar{b}$ } scalar

$\bar{a} \to \square \to \|\bar{a}\|^2$

Orthogonal basis vectors

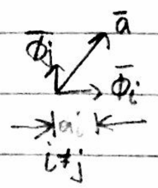
$\{\bar{\phi}_i\}_1^N : \bar{\phi}_i\cdot\bar{\phi}_j = 0,\ i \neq j$



$\bar{\phi}_i\cdot\bar{\phi}_i = 1\ \forall i$

$\{\bar{\phi}_i\}_1^N$: spans the N-dim space

↳ not unique



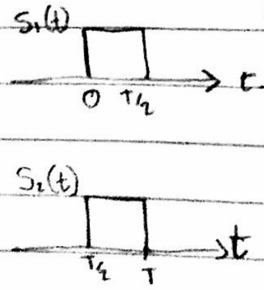
$\bar{a} = \underbrace{\sum_{i=1}^{N} a_i\bar{\phi}_i}_{\text{decomp}}$ projection of $\bar{a}$ on the axis ϕ_i

↳ $a_i = \bar{a}\cdot\bar{\phi}_i = \|\bar{a}\|\|\bar{\phi}_i\|\cos\theta$

Inner Product

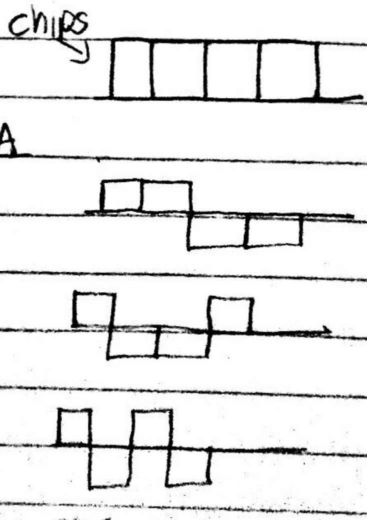
$\int s_1(t)\,s_2(t)\,dt$

$\int S_1(f)\,S_2(f)\,df$



TDMA
FDMA
$\perp$ CDMA

Ex: CDMA



Spred spectrum

3G CDMA

spreding factor → 128

↳ 128 users

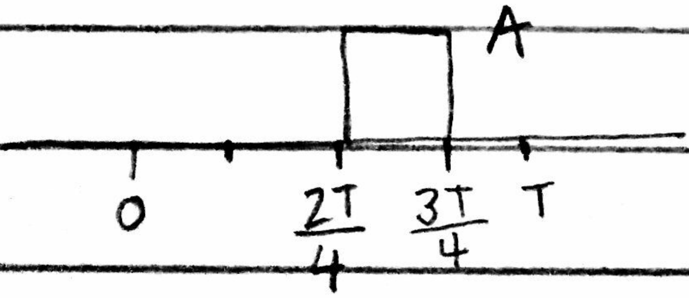
5G -> 2020s -> CDMA-OFDM hybrid (Huawei)

Energy is like the norm squared of a vector

VDSL2+ -> 2^{15}-ary signalling
QAM

$s_i(t) = A_i \cos(2\pi f_c t + \theta_i) \quad i = 1, 2, \ldots, 2^{15}$

$\phi_3(t)$



$E = A^2\, T/4 = 1$

$\therefore A = \sqrt{\frac{4}{T}}$