Lecture 18 Nov 16, 2015

If SNR is sufficiently high, there is an opportunity for m-ary signaling in a reliable way.

$R_b = (\log_2 M) R_s$

$\log_2 M$ bits of info in a symbol

<u>detection in M-ary</u>

$m_1, m_2, \ldots, m_M$

Special case: binary

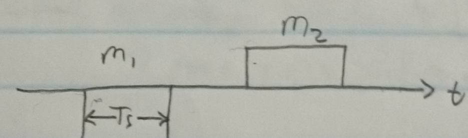
$$\begin{array}{cc} m_1 , & m_2 \\ \uparrow & \uparrow \\ -1 & 1 \end{array}$$

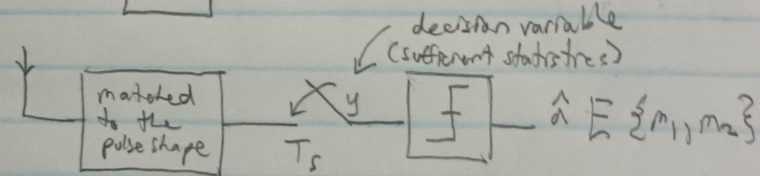
A special case: M-ary PAM

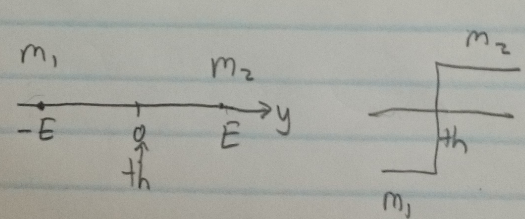
pulse-amplitude-modulation

one pulse shape; info is in the amplitude

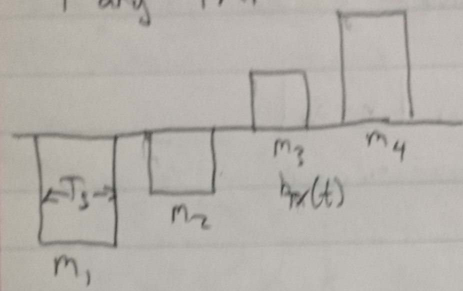
Ex:



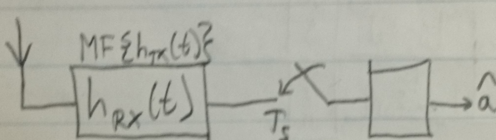


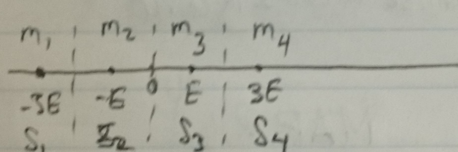


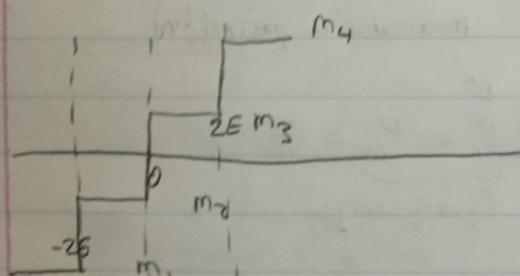
4-ary PAM



$-3h_{Tx}(t)$ $-h_{Tx}(t)$ $h_{Tx}(t)$ $3h_{Tx}(t)$



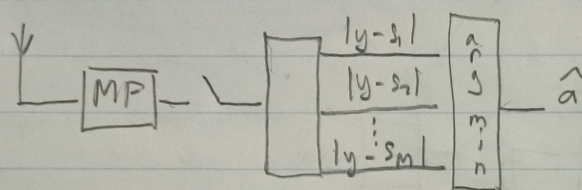




Minimum Distance Receiver

take the minimum of $|y-s_1|, \dots |y-s_4|$

$$\boxed{\hat{a} = M_{\substack{\arg\min|y-s_i| \\ i\in\{1,\dots M\}}}}$$



minimize distance receiver

$$\left.\begin{matrix} P(m_1|y) \\ \vdots \\ P(m_M|y) \end{matrix}\right\} \arg\max$$

MAP Rule

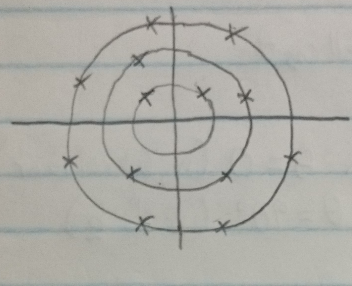
maximum a posteriori

if $\underbrace{P_1 = \cdots P_M}_{\text{Equally likely}} = \frac{1}{M}$

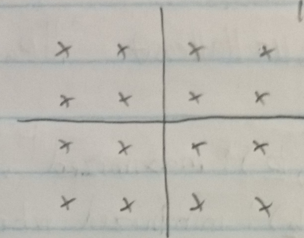
MAP rule = ML rule
(maximum likelyhood)

$$\left.\begin{matrix} f(y|m_1) \\ \vdots \\ f(y|m_M) \end{matrix}\right\} \arg\max$$ ML

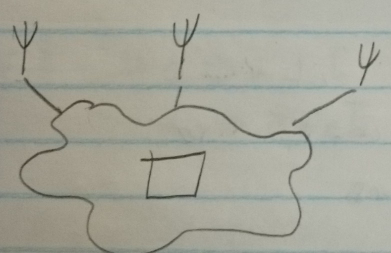
1D signaling m-ary PAM
2D signaling sinusoid info in amp + phase; M-QAM



16 QAM

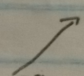


Cloud Radio Access Network



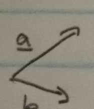
5G

Linear Algebra



$\underline{a}$

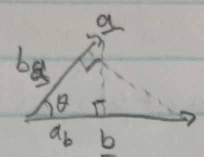
$\|a\|$: norm



$\underline{a}$ $\underline{b}$

$$(\underline{a}, \underline{b}) \triangleq \|a\| \|b\| \cos\theta = \|a\| \|b_a\| = \|b\| \|a_b\|$$

$$\theta = \arccos \frac{(\underline{a}, \underline{b})}{\|a\| \|b\|}$$

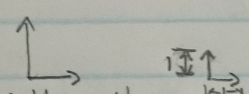


$\|a_b\| = \|a\| \cos\theta \qquad \|b_a\| = \|b\| \cos\theta$

$(\underline{a}, \underline{b})$: maximized when $\theta = 0$ (a, b = colinear)
$(\underline{a}, \underline{b})$: minimized when $\theta = 90°$ ($\underline{a} \perp \underline{b}$)

Dimension: How many basis vectors I need to represent a set of vectors? $\{\underline{a}_i\}_{i=1}^{M}$

Orthonormal: $\perp$ + $\| \ \| = 1$



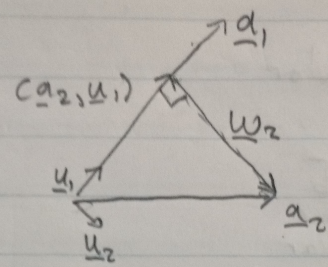
orthogonal

Dim $\Rightarrow$ 2 $\Rightarrow$ 2 basis vectors

Gram - Schmidt Orthogonalization Procedure

$$\underline{u}_1 = \frac{\underline{a}_1}{\|\underline{a}_1\|}$$

$$\underline{u}_2 = \frac{\underline{w}_2}{\|\underline{w}_2\|} \qquad \underline{w}_2 = \underline{a}_2 - \underbrace{\overbrace{(\underline{a}_2, \underline{u}_1)}^{\text{Scalar}} \underline{u}_1}_{\text{part of } \underline{a}_2 \text{ on } \underline{a}_1 \text{ direction}}$$



$$\underline{u}_i = \frac{\underline{w}_i}{\|\underline{w}_i\|} \qquad \underline{w}_i = \underline{a}_i - \sum_{j=1}^{i-1} (\underline{a}_i, \underline{u}_j)\,\underline{u}_j$$

Lecture 19 Nov. 18 2015

$\bar{a}$, $\|\bar{a}\|$, $\bar{a}\cdot\bar{b} = \|\bar{a}\|\|\bar{b}\|\cos\theta$

$$\theta = \arccos\left(\frac{\bar{a}\cdot\bar{b}}{\|\bar{a}\|\|\bar{b}\|}\right)$$

$\bar{a}\cdot\bar{b} \rightarrow 0 \Rightarrow \bar{a} \perp \bar{b}$

$\rightarrow \max \Rightarrow \bar{a} = c\bar{b}$

dot product shows how much the vectors look alike

$$\bar{a}\cdot\bar{a} = \|\bar{a}\|^2 \Rightarrow \|\bar{a}\| = \sqrt{\bar{a}\cdot\bar{a}}$$

<u>Statistics</u>		<u>Vectors</u>		<u>Waveforms (Signals)</u>
Correlation	=	dot product	=	inner product