

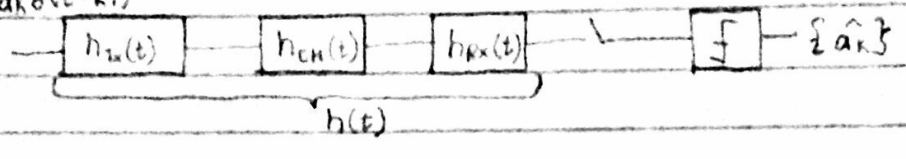
11/04/15

# Lecture 15

## ISI

"no noise"

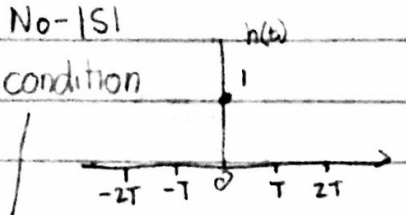
$$\sum a_k \delta(t - kT)$$



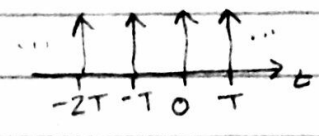
$$h(t) = \begin{cases} 1, & t=0 \\ 0, & t=kT, k \neq 0 \end{cases}$$

Best  $h(t)$ ?  
min BW

No-ISI condition



$$h(t) \sum \delta(t - kT) = \delta(t)$$



↳ Nyquist condition

→ FT

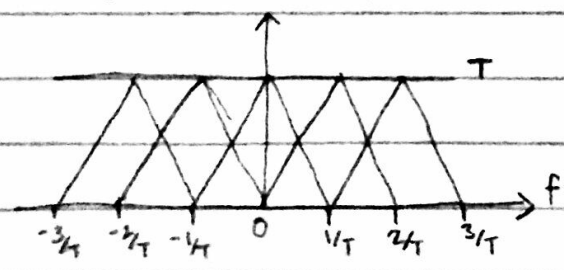
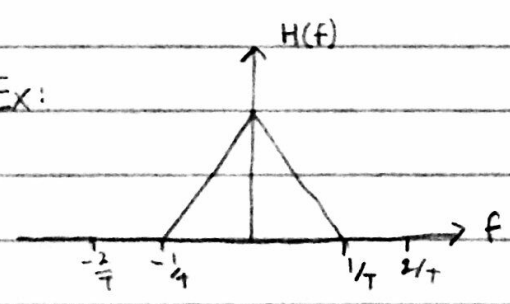
$$\text{FT}(h(t)) * \text{FT}(\sum \delta(t - kT)) = 1$$

$$\rightarrow H(f) * \frac{1}{T} \sum \delta(f - \frac{k}{T}) = 1$$

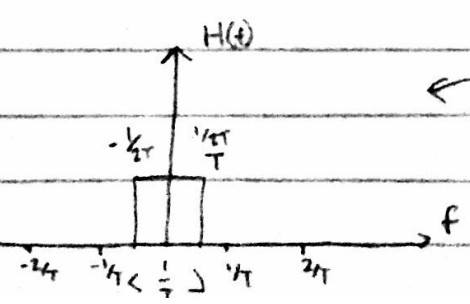
$$\rightarrow \frac{1}{T} \sum H(f - \frac{k}{T}) = 1$$

$$\sum_{-\infty}^{\infty} H(f - \frac{k}{T}) = T \quad \text{Nyquist no ISI condition}$$

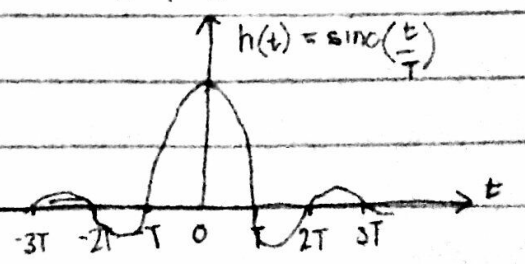
Ex:



$H(f)$  with min BW?



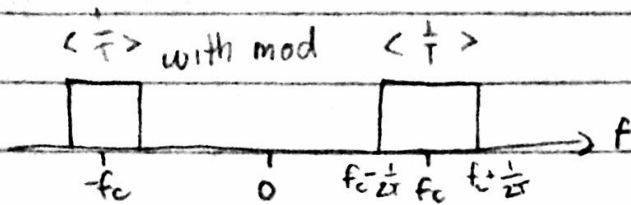
$R = \frac{1}{T}$  sym/s



$$BW = \frac{1}{2T} = \frac{R}{2}$$

$$\text{Spectral Efficiency} = \frac{R}{BW} = \frac{R}{R/2} = 2$$

$$SE_{\text{max, sym, baseband}} = 2 \text{ symbols/sec/Hz}$$



$$SE_{\text{max, sym, mod}} = 1 \text{ symbols/sec/Hz}$$

$$R = SE \times B$$

$$= \frac{\text{syms/sec}}{\text{Hz}} \times \text{Hz}$$

sym/sec

Packing Rate: What is the max # of bits you can pack in a symbol?  
bits/sym

$$64\text{QAM} \rightarrow \log_2 64 = 6 \text{ bits/sym}$$

Why not just use high M-ary all the time? Because noise

$$\text{Packing Rate} = \log_2(1 + \text{SNR}) \text{ bits/sym}$$

$$\text{Ex: SNR} = 9 \text{ dB}$$

$$\log_2(1 + 8) = \log_2 9 = 3.2 \text{ bits/sym}$$

$$SE_{\text{max}} = 1 \frac{\text{sym/sec}}{\text{Hz}} \times \log_2(1 + \text{SNR}) \frac{\text{bits}}{\text{sym}}$$

$$= \log_2(1 + \text{SNR}) \text{ bits/sec/Hz}$$

Nyquist + Shannon

$$\hookrightarrow BW \times SE_{\text{max}} = W \log_2(1 + \text{SNR}) = R_{\text{max}}$$

$$R_{\text{max}} = \underbrace{BW}_{n_T} \underbrace{SE_{\text{max}}}_{n_{R_T}} \log_2(1 + \text{SNR})$$

$$\hookrightarrow \min(n_T, n_{R_T}) = \text{mimo gain}$$

Ex: 4G LTE-Advanced

Carrier aggregation:  $5 \times 20 \text{ MHz} = 100 \text{ MHz}$

64 QAM

8-layer MIMO

$$8 \times 100 \text{ MHz} \times \overbrace{\log_2 64 \times 1 \text{ sym/sec/Hz}}^{6 \text{ bits/sec/Hz}}$$

$= 4.8 \text{ Gb/s} \leftarrow \text{peak}$

$\sim 2035-2040$ : 6G

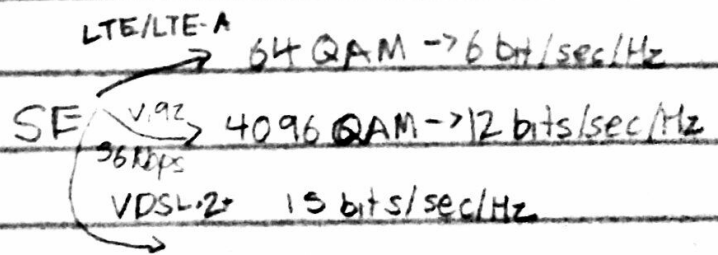
Design 1 Tbps

Find  $n, W, \text{SNR}$

such that

$$nW \log_2(1 + \text{SNR}) = 12 \text{ Tbps}$$

$\downarrow \quad \downarrow \quad \downarrow$   
 $100 \quad 100 \text{ MHz} \quad 12$   
 $[80906 \text{ Hz}] \quad \uparrow \quad 4096 \text{ QAM}$



fiber: binary: 1 bit/sec/Hz