Lecture 13

10/19/15

- Optimum (good) design
- Performance

Optimum Receiver

- with what? -> Maximize SNR
- under what conditions? -> AWGN channel

$\hookrightarrow \underbrace{\text{ideal ch}}_{\text{no distortion}} + \text{AWGN}$

for a given $\underbrace{h_{Tx}(t)}_{\text{transmit filter}}$

$h_{Rx}(t) = h_{Tx}(T-t)$

optimal

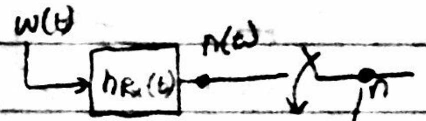
$$|H_{Rx}(f)| = |H_{Tx}(f)|$$

Term Exam

- 80 mins
- cheat sheet (double sided)

$$f_z(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-m)^2}{2\sigma^2}}$$

$P_e = BER$



$\hookrightarrow G(0, \frac{N_0 E}{2})$

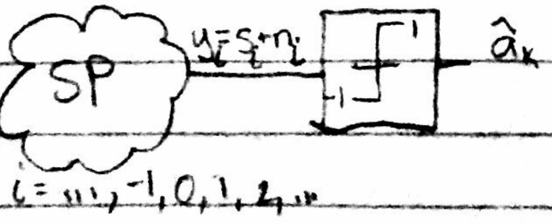
$$f_N(n) = \frac{1}{\sqrt{2\pi \cdot \frac{N_0 E}{2}}} e^{-\frac{n^2}{2\frac{N_0 E}{2}}} = \frac{1}{\sqrt{\pi N_0 E}} e^{-n^2/N_0 E}$$

$$\sigma_n^2 \triangleq E[n^2] - \cancel{E^2[n]}^{0}$$
$$= E[n(t)n(t+\tau)]|_{\tau=0}$$
$$= R_N(\tau)|_{\tau=0}$$
$$= R_N(0) = \int_{-\infty}^{\infty} S_N(f)df$$

$\hookrightarrow \underbrace{S_w(f)}_{N_0/2} |H_{Rx}(f)|^2$

$$= \frac{N_0}{2} \underbrace{\int_{-\infty}^{\infty} |H_{Rx}(f)|^2 df}_{\int_{-\infty}^{\infty} h_{Tx}(t)dt}, \quad = \int |H_{Tx}(f)|^2 df = E$$

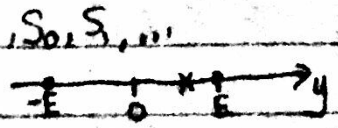
$$= \frac{N_0}{2} E$$

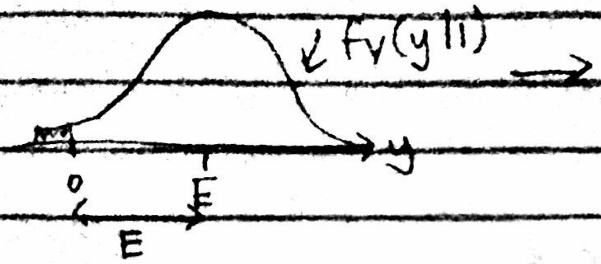


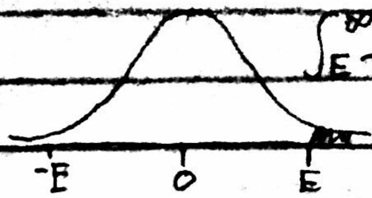
assume no noise

$y_i = \dots, s_{-1}, s_0, s_1, \dots$

$\pm E$

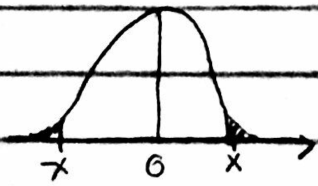






$\int_E^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2/2\sigma^2} dy$

$$G(0;\sigma^2=1/2) = \frac{1}{\sqrt{2\pi \frac{1}{2}}} e^{\frac{-z^2}{2 \cdot \frac{1}{2}}} = \frac{1}{\sqrt{\pi}} e^{-z^2}$$



$$\operatorname{erfc}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-z^2} dz$$

Derivation:

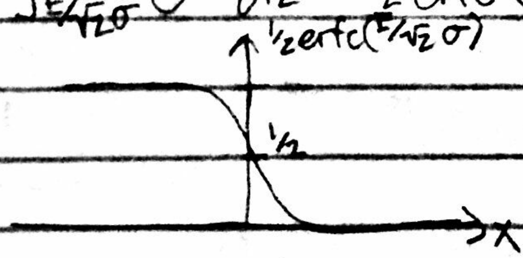
$$\int_E^{\infty} 1/\sqrt{2\pi}\sigma \, e^{-y^2/2\sigma^2} dy$$

$$z^2 = y^2/2\sigma^2$$
$$z = y/\sqrt{2}\sigma$$
$$y = \sqrt{2}\sigma z$$

$$\int_{E/\sqrt{2}\sigma}^{\infty} \frac{1}{\sqrt{2\pi}\cancel{\sigma}} e^{-z^2} \cancel{\sqrt{2}}\cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{E/\sqrt{2}\sigma}^{\infty} e^{-z^2} dz = \frac{1}{2}\operatorname{erfc}\left(\frac{E}{\sqrt{2}\sigma}\right) \quad \sigma = \sqrt{\frac{N_0 E}{2}} \longrightarrow = \sqrt{\frac{E}{N_0}}$$

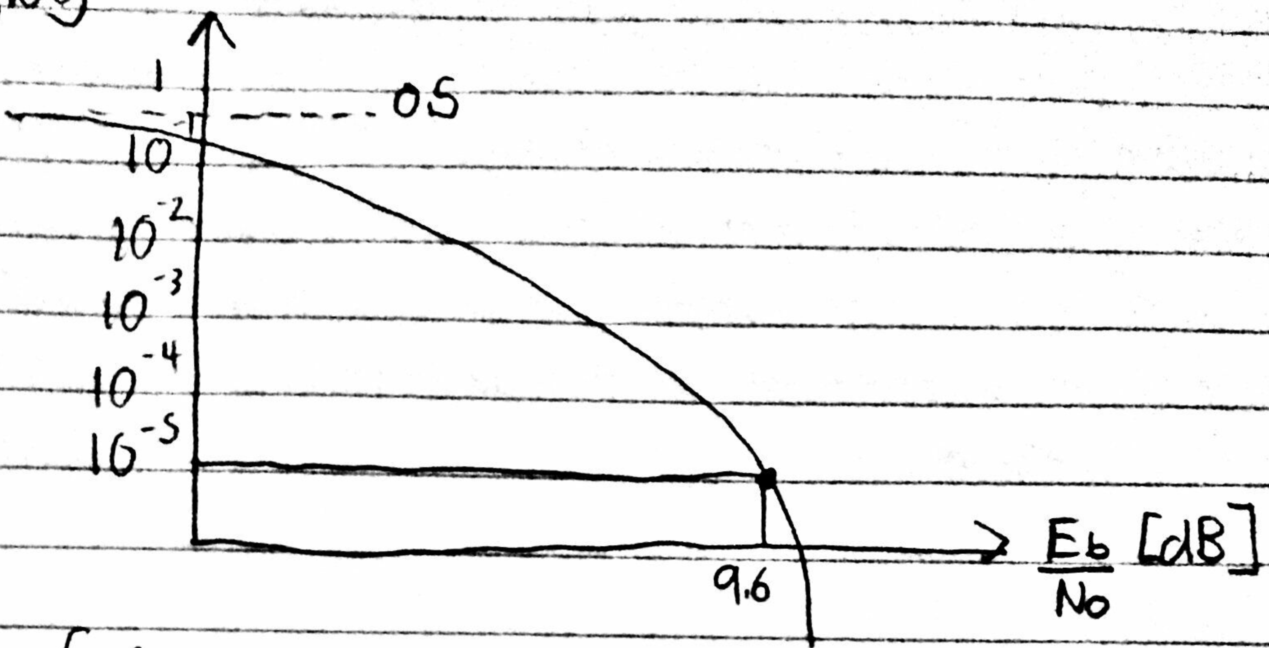
cdf



$$P_{e|1} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\right) = P_{e|-1}$$

$$P_e = P_{e|1}P_1 + P_{e|-1}P_{-1}$$
$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\right)$$

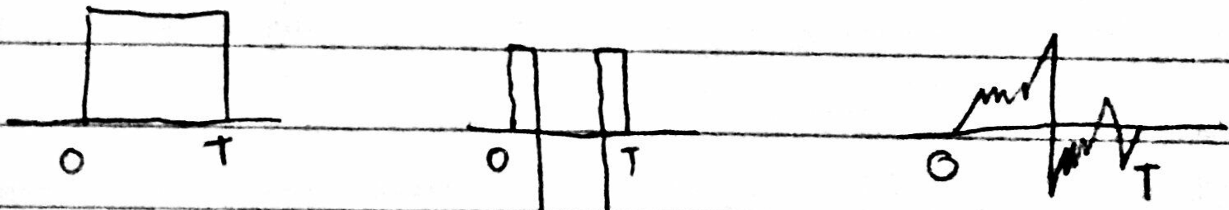
loglog



$E = \int h^2_{Tx}(t)\,dt$

Design I II III

$h_{Tx}(t)$



Assume $E_{I} = E_{II} = E_{III}$

$\therefore P_{e,I} = P_{e,II} = P_{e,III}$

but $B_{I} < B_{II} < B_{III}$ (Bandwidth)