10/14/15

Lecture 12

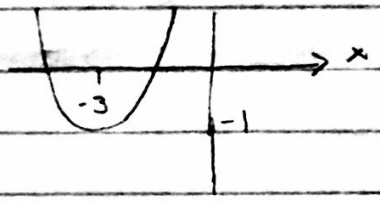
Term Exam
- Matched Filter, P_e (*)
- LTI, PSD
- dB, Power, BW

Optimization

maximize or minimize $\square$

$\square$

subject to constants



$$\min_{x} \quad (x+3)^2 - 1 = x^2 + 6x + 8$$

$$\text{s.t.} \quad x \in [-8, 0]$$

(A): -1, @ $x = -3$ (optimal)

Our case: maximize Quality of Experience subject to Power, link, BW, ...

m.o.o → multiple objective optimization

maximize SNR = $\frac{\text{instantaneous signal power}}{\text{average noise power}}$ ← maximize / ← minimize

$h_{Rx}(t)$

S.t. $h_{Tx}(t)$: given

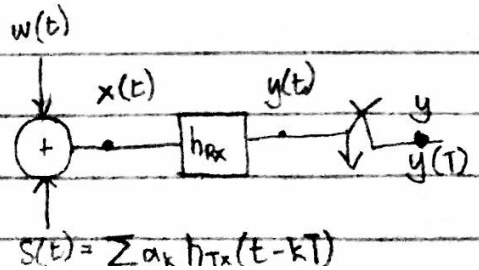
MF

$$h_{Rx}(t) = h_{Tx}(T-t)$$

$$|H_{Rx}(f)| = |H_{Tx}(f)|$$

Random signals → LTI $h(t)$ → $y(t)$

$S_x(f)$ $\qquad$ $S_y(f) = S_x(f)|H(f)|^2$

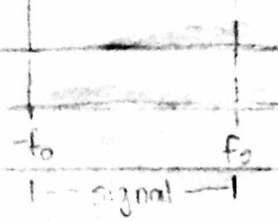


$$y(t) = [s(t) * h_{Rx}(t)] + [w(t) * h_{Rx}(t)]$$

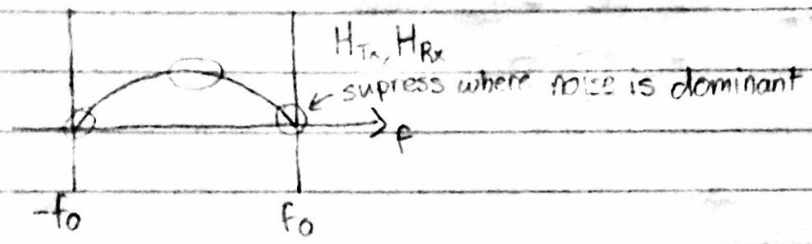
$S_s(f)|H_{Rx}(f)|^2$ $\qquad$ $S_w(f)|H_{Rx}(f)|^2$

$= \frac{|H_{Tx}(f)|^2 |H_{Rx}(f)|^2}{T} = \frac{N_0}{2}|H_{Rx}(f)|^2$

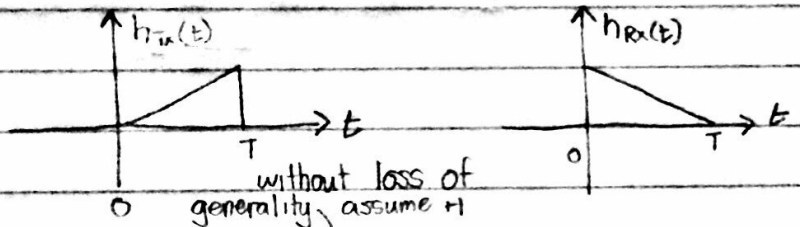
Assume $h_{Tx}(t)$ not known — accept everything in signal B.W., reject anything outside



Assume $h_{Tx}(t)$ known



$$h_{Rx}(t) = h_{Tx}(T-t)$$



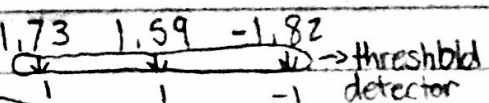
without loss of generality, assume +1

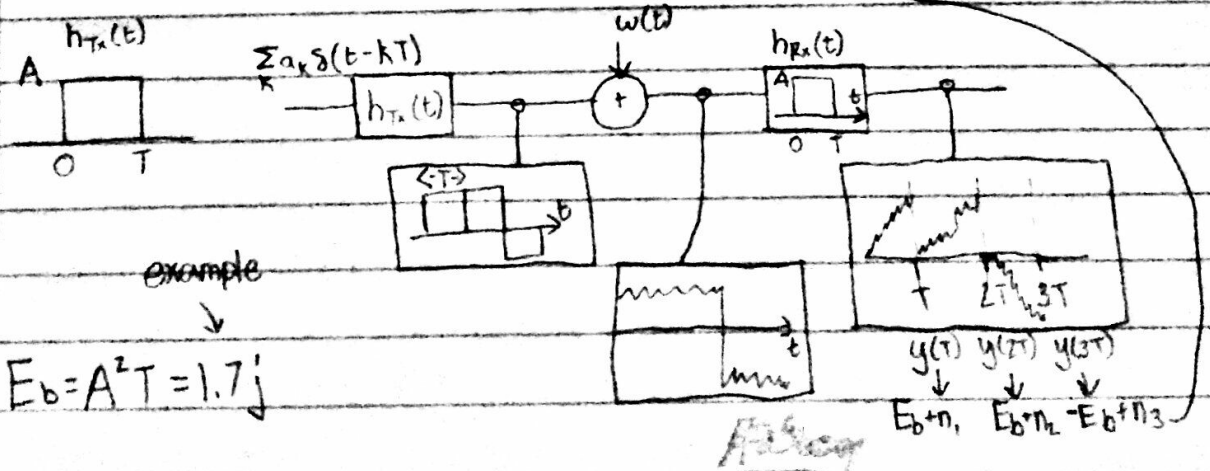
$$y = y(t)\Big|_{t=T} = y(T) = h_{Tx}(t) * h_{Rx}(t)\Big|_{t=T} + \cdots$$

$$S(t)\Big|_{t=T} = S(T) = S$$

$$S = \int_{\tau} h_{Tx}(\tau)\underbrace{h_{Rx}(t-\tau)}_{h_{Tx}(\tau)}d\tau$$

$S = \int h_{Tx}^2(t)dt = E_b$ (bit Energy)

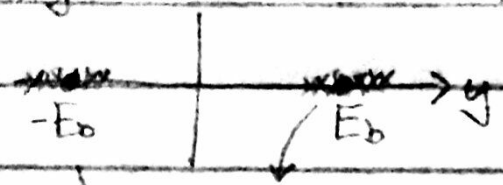


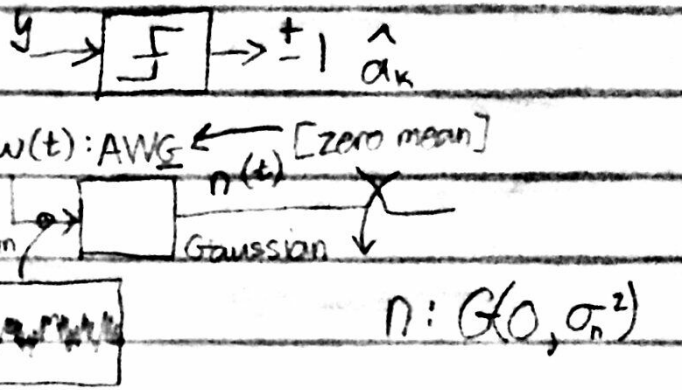


example

$E_b = A^2T = 1.7$ J

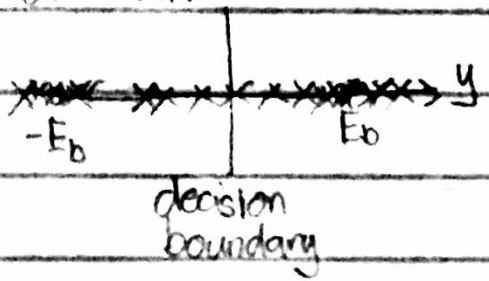
high SNR constellation





$n : G(0, \sigma_n^2)$

low SNR



Assume 1 transmitted

If $n < -E_b$

→ error

$$\sigma_n^2 \triangleq E[n^2] - E^2[n]$$

$$= E[n^2]$$

$$= R_n(0) = \int_{-\infty}^{\infty} S_n(f)df = \int S_w(f)|H_{Rx}(f)|^2$$

$$= \frac{N_0}{2}\int |H_{Rx}(f)|^2 df = \frac{N_0}{2}\int |h_{Rx}(t)|^2 dt$$

$$= \frac{N_0}{2} E_b$$