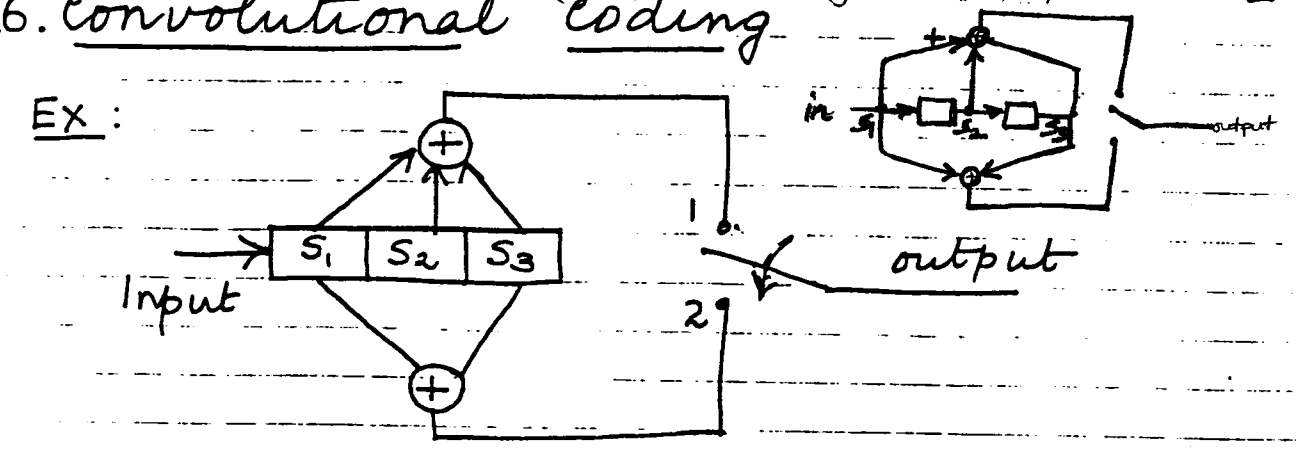
26. Convolutional coding

Ex:



K = 3 stage SR. $\nu = 2$ bits out for every bit in.

R bit/s input data stream
$\Rightarrow$ $R\nu$ bit/s coded output

$c_1 = s_1 \oplus s_2 \oplus s_3$

$c_2 = s_1 \oplus s_3$

Time interval →	1	2	3	4	5	6	7	8
Input bits	0	1	1	0	1	0	0	1
Output bits	00	11	01	01	00	10	11	11

(00 11: initialization)

- K = constraint length
  Rate 1/2 coder (1/ν coder)
- In general,

$$c_k = \sum_{j=1}^{K} h_{kj} s_j \pmod 2 \qquad k = 1, \ldots, \nu$$

h_{ij}'s are 1 or 0.

$\Rightarrow$ convolutional coder

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Time interval	S_1	S_2	S_3	c_1 $(s_1 \oplus s_2 \oplus s_3)$	c_2 $(s_1 \oplus s_3)$
	0	0	0		
1	[0]	0	0	0	0
2	[1]	0	0	1	1
3	[1]	1	0	0	1
4	[0]	1	1	0	1
5	[1]	0	1	0	0
6	[0]	1	0	1	0
7	[0]	0	1	1	1
8	[1]	0	0	1	1

- Add K zeros to clear the SR

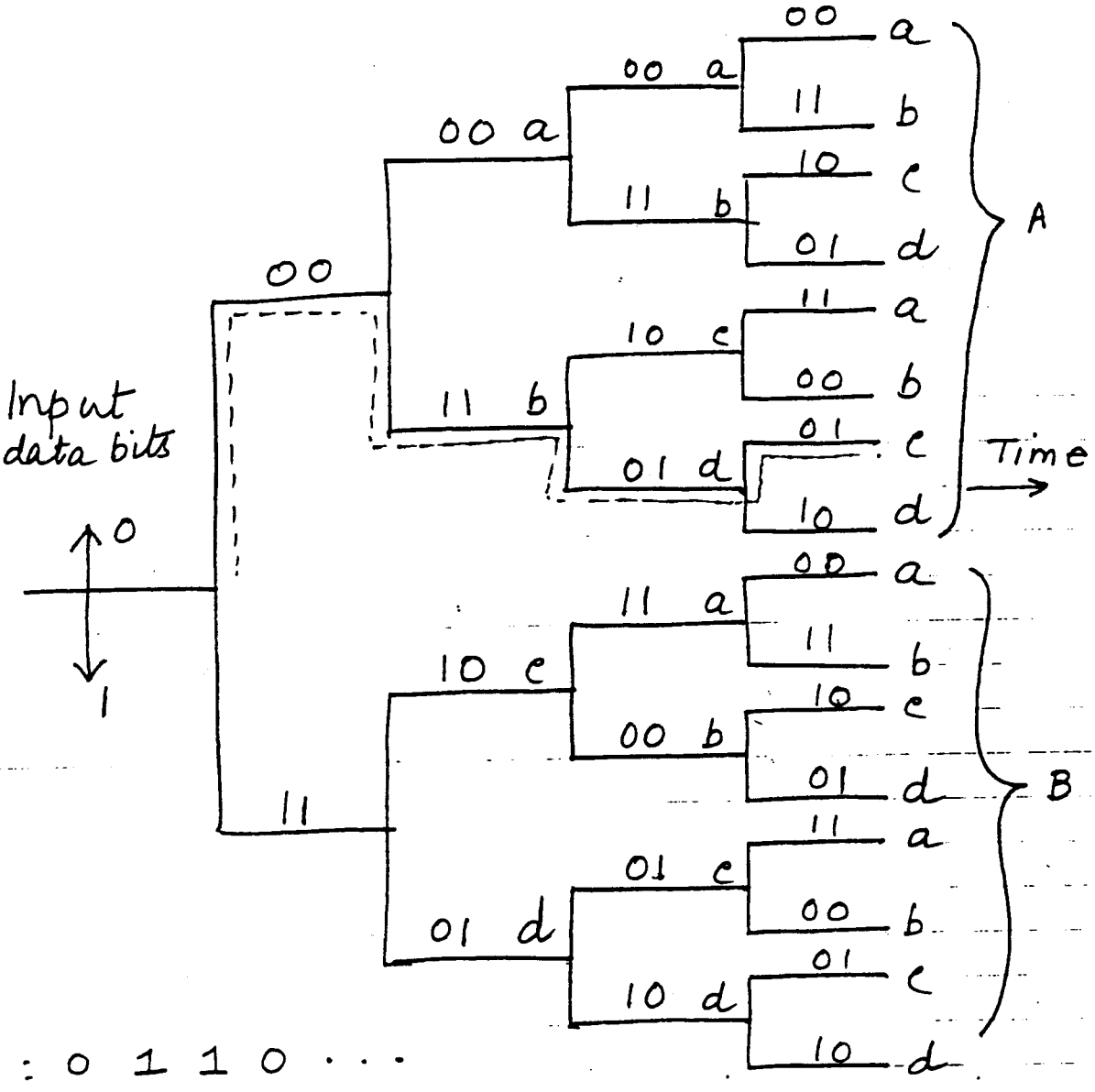
0	1	0	1	0
0	0	1	1	1
0	0	0	0	0

- For 'k' data digits, we get
  $n = (K + k)v$ digits at output

In practice $k >> K$, hence

k digits in $\Rightarrow \cong$ k v digits out

- unlike block coder, conv. coder operates on a continuous basis



. Input : 0 1 1 0 ...

Tree Representation

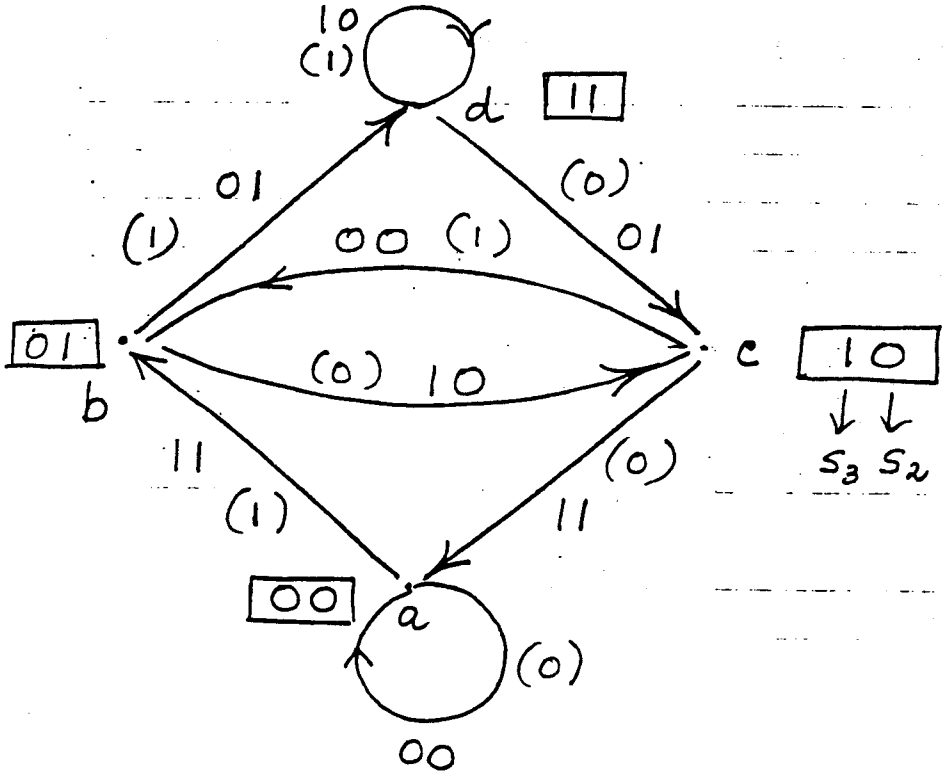
. cyclic nature of the coder (after the 3rd branch) : A & B are identical

. no. of possible sequences increases exponentially with time

. Hamming distance between paths increases
⇒ basis of improved performance of coder

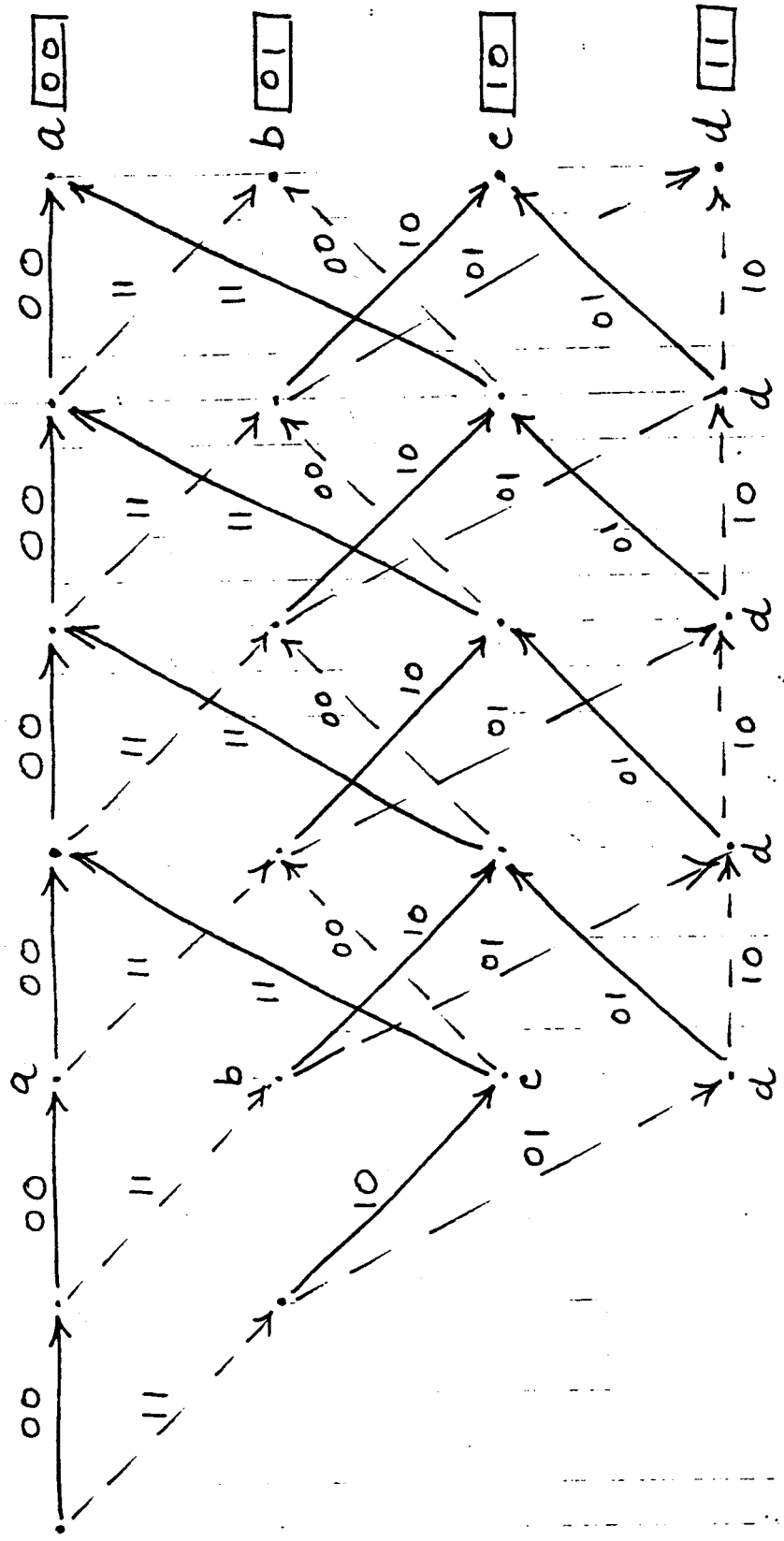
. finite memory ⇒ $0\,x_1\,x_2\,x_3\,x_4$ ... and $1\,x_1\,x_2\,x_3\,x_4$ ... generate same output after 3rd group of output bits

- After initialization, output bits are determined by the $(K-1) = 2$ stage S_3, S_2 bits + new data bit arriving to fill the stage S_1
- e.g. in interval 3, $[S_3, S_2] = [0\ 1]$
  a 1 arriving in interval 3 produces 01.
  If 0 arrives (e.g. interval 6), output is 10.
- 2^{K-1} states. input bit determines the transitions



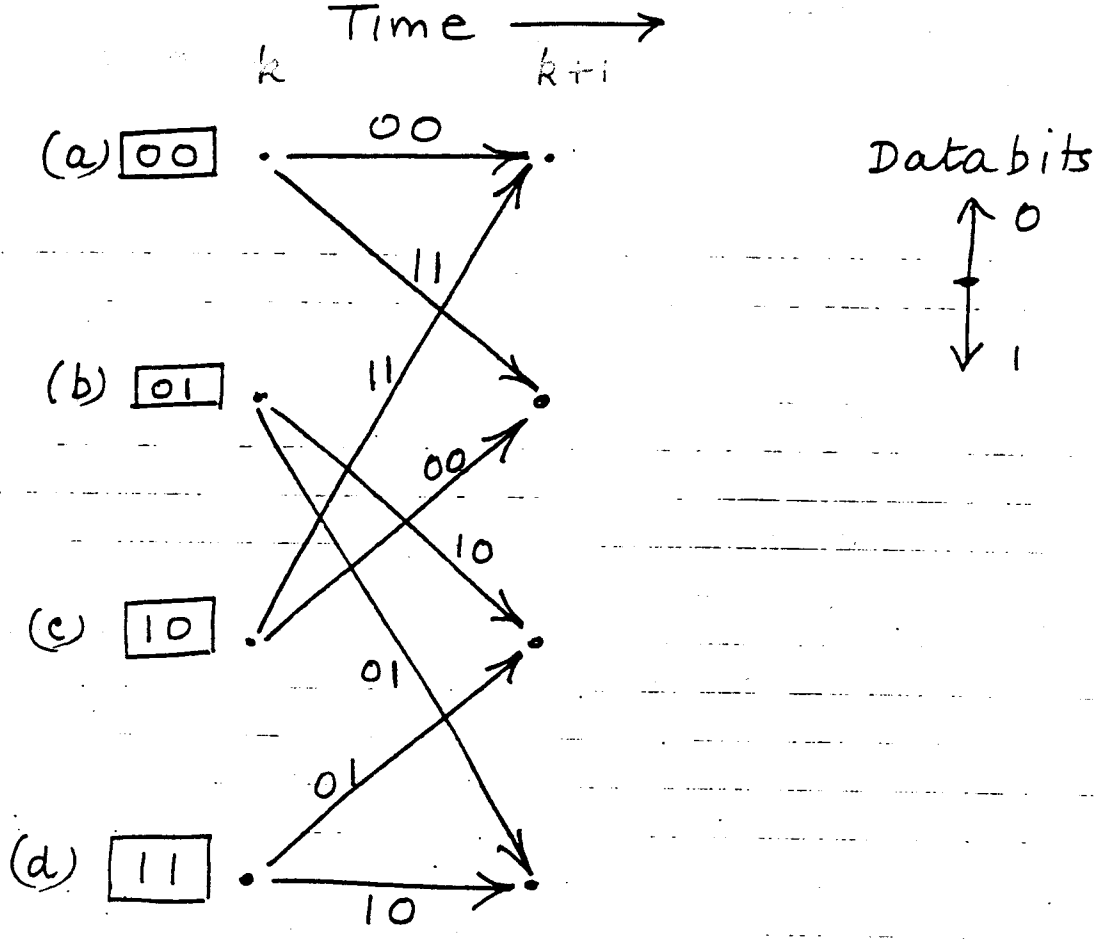
state diagram

- Four states, a, b, c, d
- Can all $K = 3$, rate 1/2 coders be represented by above diagram?



Trellis Diagram

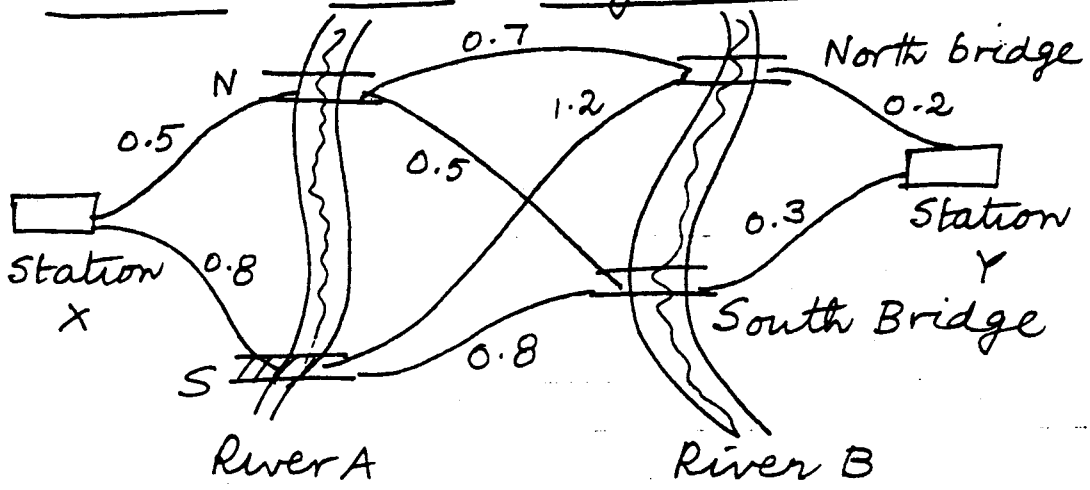
solid line → 0 dotted line → 1



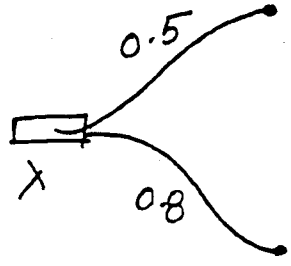
Trellis Representation

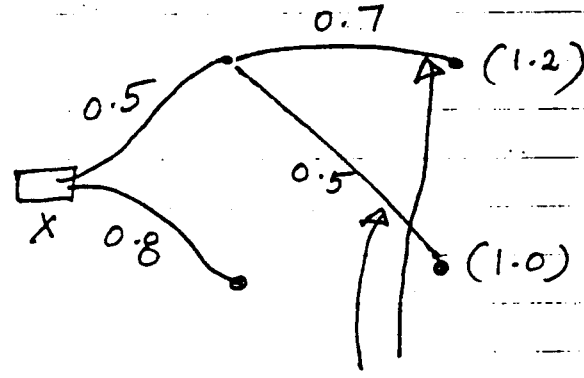
- shows the repetitive nature of the finite-state machine
- optimum decoding
  - ⇒ max likelihood decoding
  - ⇒ choose the path closest to the received sequence in the Hamming distance sense.
- Viterbi algorithm
- dynamic programming

Shortest Path Algorithm

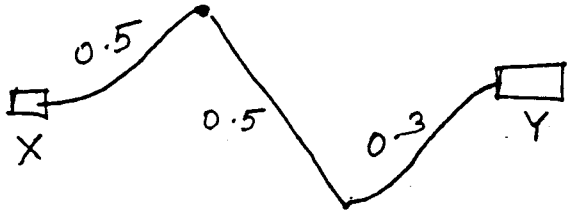


- Shortest path from X to Y ?
- Optimum total path must lie along the optimum path from X to either the north or south bridge across River B





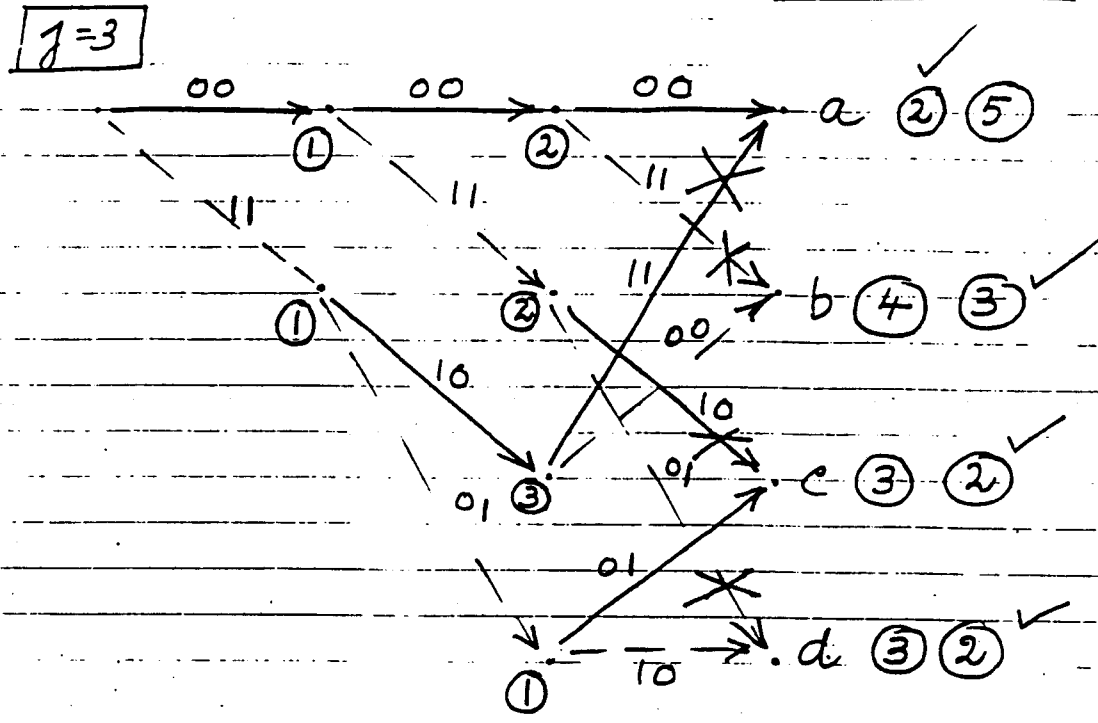
survivors



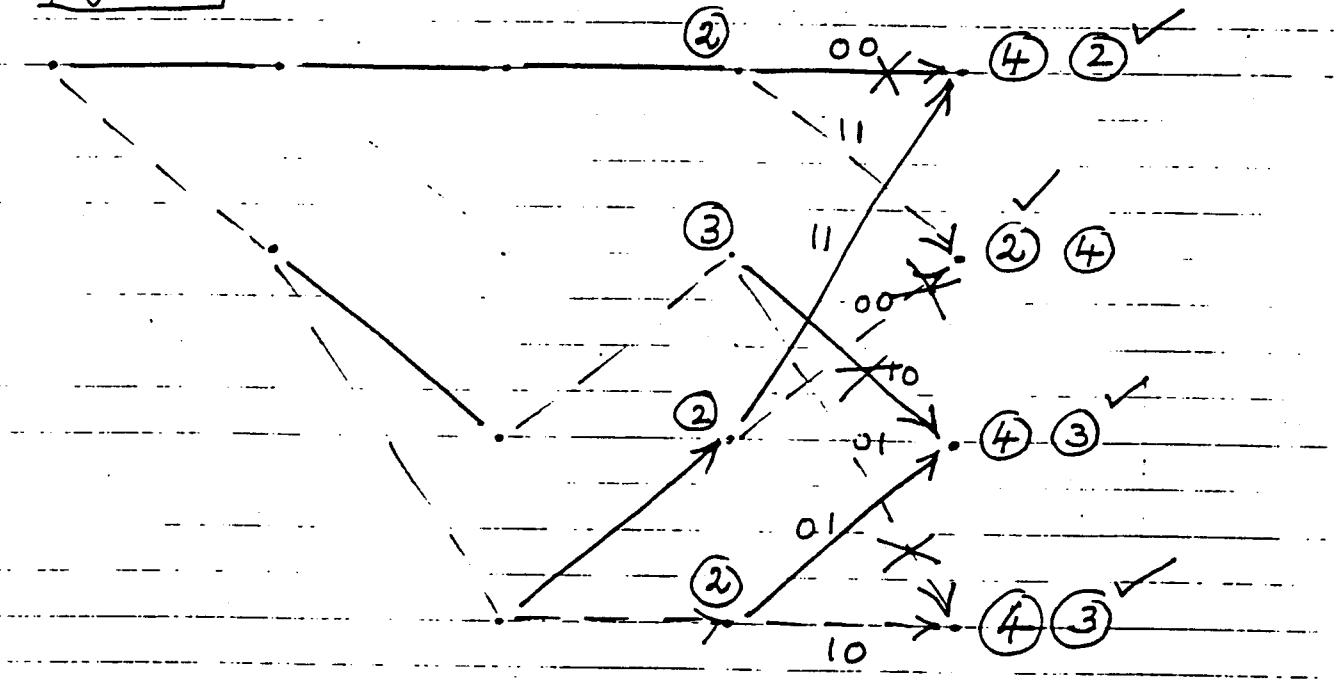
- If N rivers ⇒ $4(N-1)+2$ additions by dynamic programming
  → $(N-1)2^N$ additions by brute force
- Add, compare and select

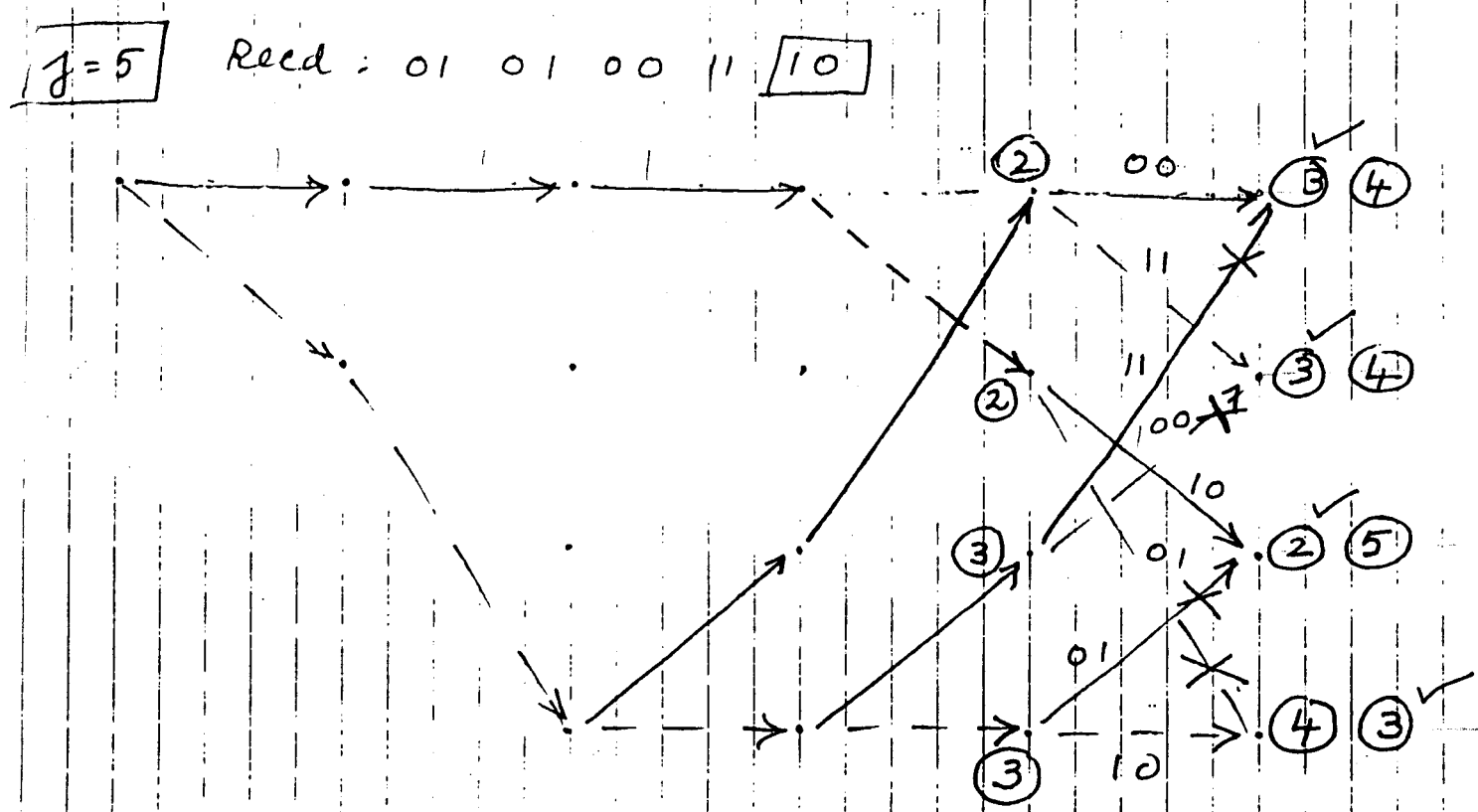
EX: received sequence: 01 01 00 11 10 ...

$j=3$



$j=4$ 01 01 00 11 10

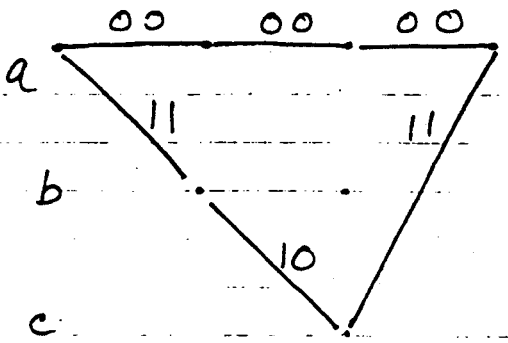




- Path memory $\simeq$ 4K or 5K is sufficient
  $\Rightarrow$ 'oldest' bit can be decoded
  e.g. if path memory = 5, survivor with smallest distance at interval 5 $\Rightarrow$ one with distance 2 $\Rightarrow$ can decode 1 bit as '0'.

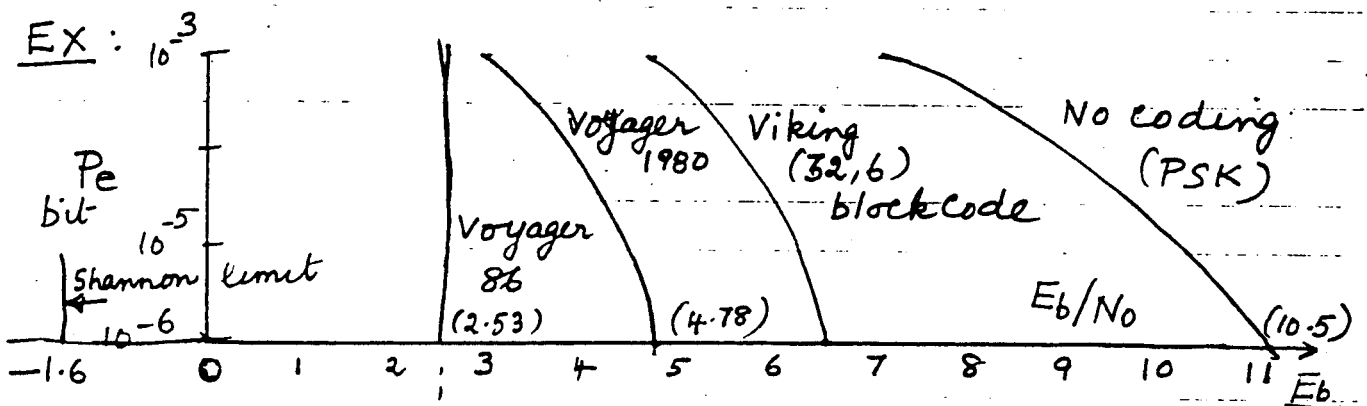
Performance of convolutional coders

- Error occurs when selected path deviates from the correct path
- If all paths are equally likely, can choose 00 ... 00 path as correct



Path with min. Hamming distance (5)
⇒ min. free distance d_f

- The larger the free distance, the better the performance
- d_f increases with constraint length, K



- Voyager (80) → K = 7 rate = ½ conv. code
- Voyager (86) → conv. coding + (255, 223) RS code
  (7, ½) (Viterbi decoding) (8 bit symbol)
  ⇒ 8-dB advantage at 10^{-6} over uncoded PSK

(power limited systems)