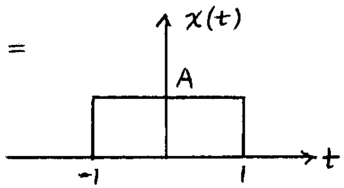
5

Review

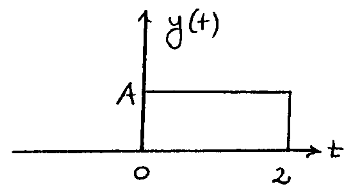
Convolution: $x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$

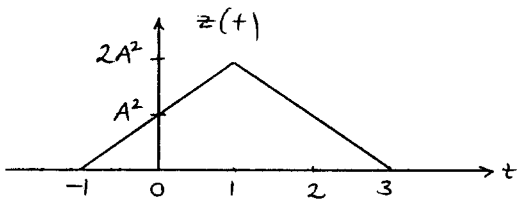
$= \int_{-\infty}^{\infty} x(t-\tau)\, y(\tau)\, d\tau$

$z(t) =$



$*$





(see exercise #1 in the web)

$z(t) = x(t) * y(t)$

$Z(f) = X(f)\, Y(f)$

Delta Function

$\delta(t) = 0, \quad \forall t$ except $t \neq 0$, and $\int_{-\infty}^{\infty} \delta(t)\, dt = \int_{0^-}^{0^+} \delta(t)\, dt = 1$

$g(t)\, \delta(t) = g(0)\, \delta(t)$

$g(t)\, \delta(t-t_0) = g(t_0)\, \delta(t-t_0)$

$g(t) * \delta(t) = \int_{-\infty}^{\infty} g(\tau)\, \delta(t-\tau)\, d\tau = \int_{-\infty}^{\infty} g(t)\, \delta(t-\tau)\, d\tau$

$= g(t)$

6

Fourier Transform

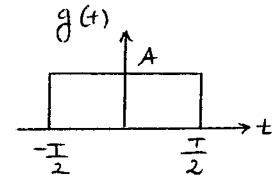
$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$

$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$

Ex: $g(t) = A\, \mathrm{rect}(t/T)$, $\quad G(f) = ?$

$\mathrm{rect}(t) = \begin{cases} 1, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & \text{else} \end{cases}$

FT pairs



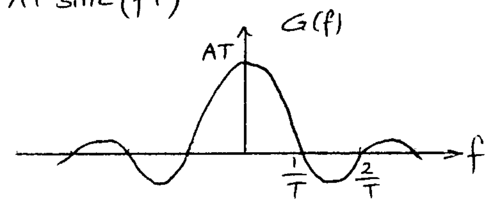
$G(f) = A \int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t}{T}\right) e^{-j2\pi f t}\, dt = A \int_{-T/2}^{T/2} e^{-j2\pi f t}\, dt$

$= A \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-T/2}^{T/2} = \frac{A}{-j2\pi f} \left(e^{-j2\pi f \frac{T}{2}} - e^{j2\pi f \frac{T}{2}} \right)$

$= \frac{A}{\pi f} \frac{1}{j2} \left(e^{j2\pi f \frac{T}{2}} - e^{-j2\pi f \frac{T}{2}} \right)$

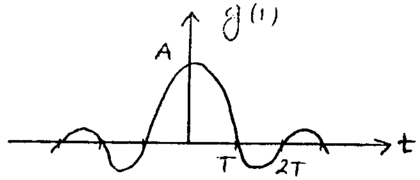
$= \frac{A}{\pi f} \sin(\pi f T) \qquad \mathrm{sinc}\, x \triangleq \frac{\sin \pi x}{\pi x}$

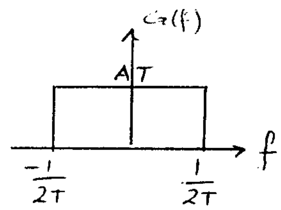
$= AT\, \mathrm{sinc}(fT)$



shortcut:
1. rect $\leftrightarrow$ sinc
2. Area $\rightarrow$ value at origin
3. value at origin $\leftarrow$ (value at origin) $\times$ (first zero-crossing)

7





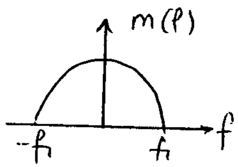
$$A\,\mathrm{sinc}(t/T) \longleftrightarrow AT\,\mathrm{rect}(fT)$$

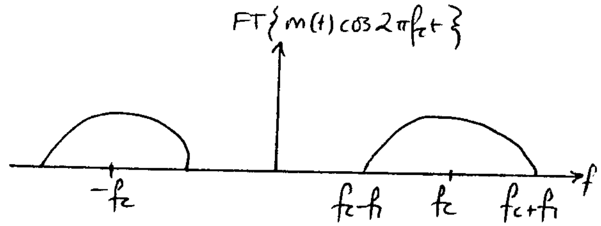
1 bit$/T$ secs $\rightarrow$ Bitrate $= R = \frac{1}{T}$ bps

$$R \propto BW$$

$$\boxed{\text{Pulse duration} \xleftrightarrow[\text{relation}]{\text{inverse}} \text{BW requirement}}$$

<u>Bandwidth</u> : extent of significant spectral content of the signal for $\boxed{+ve}$ frequencies.



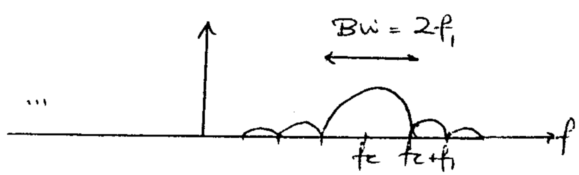


$BW(m(t)) = f_1$

$$BW\{m(t)\cos(2\pi f_c t)\} = 2f_1$$

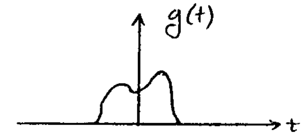
* various BW definitions

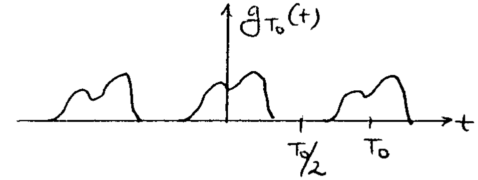
One definition : null-to-null BW



8

Fourier Transform of Periodic Signals





$$g_{T_0}(t) = \sum_{m=-\infty}^{\infty} g(t - mT_0)$$

$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$ ... complex exponential Fourier Series

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi n f_0 t}\, dt$$

$$c_n = f_0 \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi n f_0 t}\, dt = f_0\, G(nf_0)$$

$$\therefore \quad g_{T_0}(t) = \sum_m g(t - mT_0) = f_0 \sum_n G(nf_0)\, e^{j2\pi n f_0 t}$$

Poisson's sum

$$\Rightarrow \quad \sum g(t - mT_0) \longleftrightarrow f_0 \sum_n G(nf_0)\,\delta(f - nf_0)$$