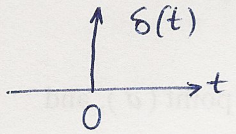
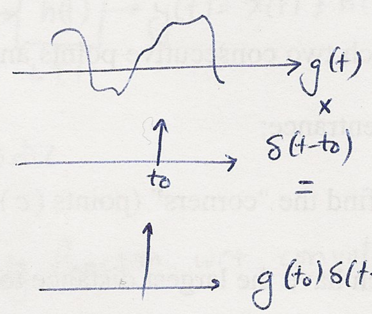
Delta Function (Dirac Delta Function)



$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \int_{0^-}^{0^+} \delta(t)\,dt = 1 \end{cases} \rightarrow \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$g(t)\,\delta(t) = g(0)\,\delta(t)$$

$$g(t)\,\delta(t-t_0) = g(t_0)\,\delta(t-t_0)$$



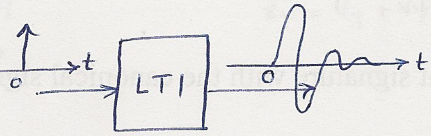
Linear Time-Invariant Systems

linear $\begin{cases} x(t) \rightarrow y(t) \\ a_1 x_1(t) + a_2 x_2(t) \rightarrow b_1 y_1(t) + b_2 y_2(t) \end{cases}$

$\begin{cases} x(t) \rightarrow y(t) \\ x(t-t_1) \rightarrow y(t-t_1) \end{cases}$ time-invariant

Impulse Response

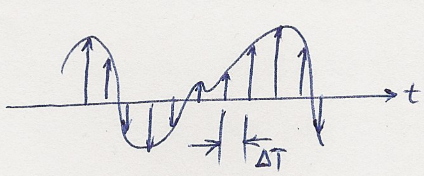
$\delta(t) \rightarrow$ [LTI] $\rightarrow h(t)$



$x(t) \rightarrow$ [$h(t)$] $\rightarrow y(t)$? (response to an arbitrary input)

If I can write $x(t)$ as a weighted sum of shifted delta functions, then the output will be weighted sum of shifted impulse responses!

An alternative representation of $x(t)$: $x(t) = \lim_{\Delta T \to 0} \sum_n x(n\Delta T)\,\delta(t - n\Delta T)\,\Delta T$



$$x(t) = \lim_{\Delta T \to 0} \sum_{n=-\infty}^{\infty} x(n\Delta T)\,\delta(t - n\Delta T)\,\Delta T \rightarrow \boxed{LTI} \rightarrow y(t) = \lim_{\Delta T \to 0} \sum_{n=-\infty}^{\infty} x(n\Delta T)\,h(t - n\Delta T)\,\Delta T$$

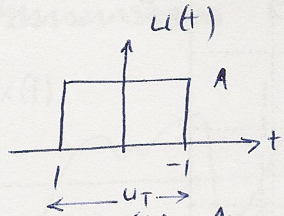
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \rightarrow \underset{h(t)}{\boxed{LTI}} \rightarrow \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

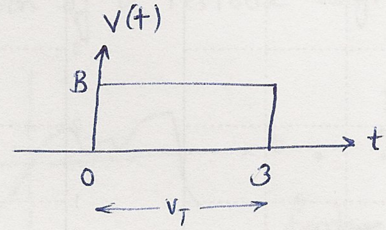
$$x(t) \rightarrow \underset{LTI}{\boxed{h(t)}} \rightarrow y(t) = x(t) * h(t)$$

Convolution: $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

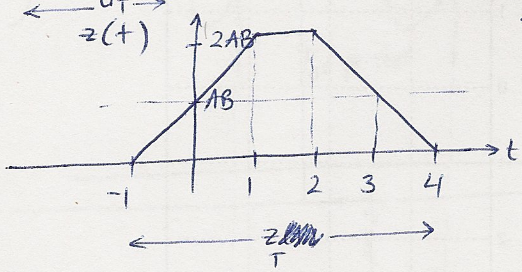
$x(t) * \delta(t) = x(t) \rightarrow \delta(t)$ is identity function wrt convolution

Ex:





$z(t) = u(t) * v(t)$



note the broadening!

$z_T = u_T + v_T$

Convolution operation lets us to find the output for an arbitrary input, but it is a tedious, integral operation.

Is there a more compact way of I/O representation?

Perhaps, in terms of multiplication?

FOURIER TRANSFORM

$\cos 2\pi f_0 t \rightarrow$ [LTI] $\rightarrow \alpha \cos(2\pi f_0 t + \phi)$ — a pure tone goes in, a pure tone comes out (scaled and phase-shifted)

sinusoidals: the only type of functions which preserve their shape when passed through any LTI system!

$e^{j2\pi f_0 t} \rightarrow$ [LTI] $\rightarrow e^{j2\pi f_0 t} \underbrace{\alpha e^{j\phi}}_{}$

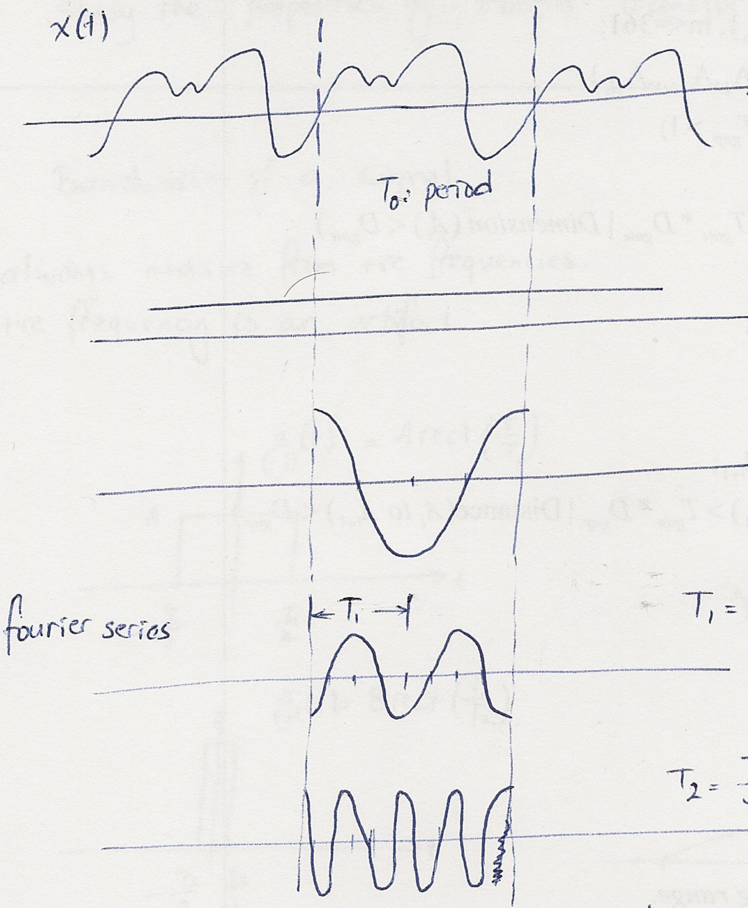
complex scaling, call it c. c is a function of f_0, and $h(t)$

Can I write an arbitrary function, in terms of sinusoids?

If yes, we can use some sort of multiplication instead of convolution.

Frequency-Domain Representation of a Periodic Signal

$x(t)$



fundamental frequency: $f_0 = \frac{1}{T_0}$

DC $(f=0)$

T_0, f_0

fourier series

$T_1 = \frac{T_0}{2}, \quad f_1 = 2f_0$

$T_2 = \frac{T_0}{3}, \quad f_2 = 3f_0$

in general, countable infinite number of harmonics.

if no edges, may be finite # of harmonics

if sharp edges, infinite series

Non-Periodic Signal → uncountable infinite number of tones

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$$

What is the proportional weight at each frequency?

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$e^{j2\pi ft} = \cos 2\pi ft + j\sin 2\pi ft$$
$$e^{-j2\pi ft} = \cos 2\pi ft - j\sin 2\pi ft$$
$$\frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2} = \cos 2\pi ft$$

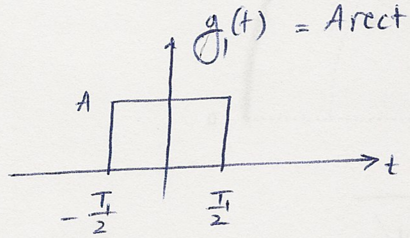
Note: one-to-one relation
for every time waveform, there is a unique freq-domain representation
for every freq-domain representation, there is a unique time waveform

* Study the properties of Fourier Transforms

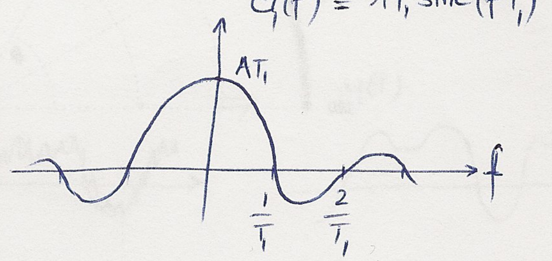
Bandwidth of a signal

* always measure from +ve frequencies.
-ve frequency is an artifact

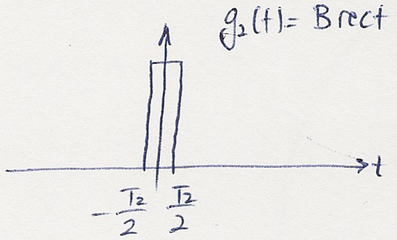
$g_1(t) = A\,\text{rect}\left(\frac{t}{T_1}\right)$



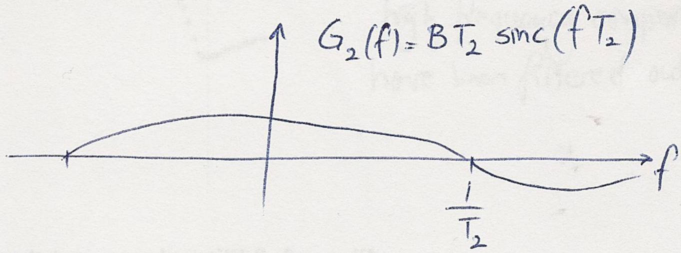
$G(f) = AT_1\,\text{sinc}(fT_1)$



$g_2(t) = B\,\text{rect}\left(\frac{t}{T_2}\right)$



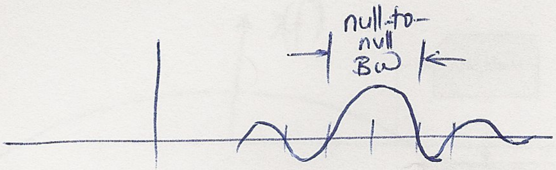
$G_2(f) = BT_2\,\text{sinc}(fT_2)$



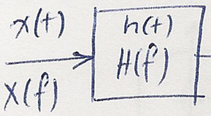
inverse relation between time and frequency.

low rate signals (wide pulses) → low BW
high rate signals (narrow pulses) → high BW

* Absolute BW
* null-to-null BW
* 95 % BW — Find f_{95} such that $\int_{-f_{95}}^{f_{95}} |X(f)|^2 df = 0.95 \int_{-\infty}^{\infty} |X(f)|^2 df$



LTI Systems



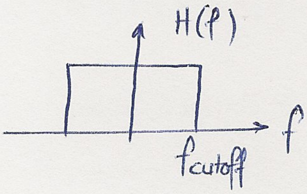
$y(t) = x(t) * h(t)$

$Y(f) = X(f)\,H(f)$

h(t): impulse response

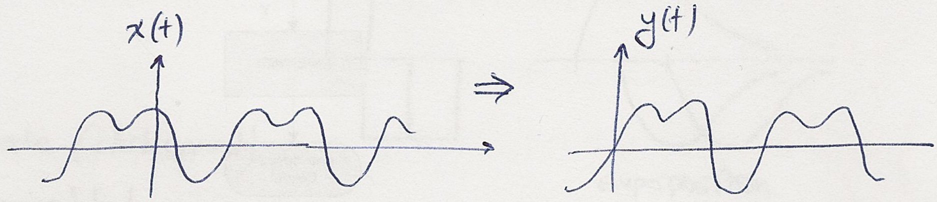
H(f): frequency response

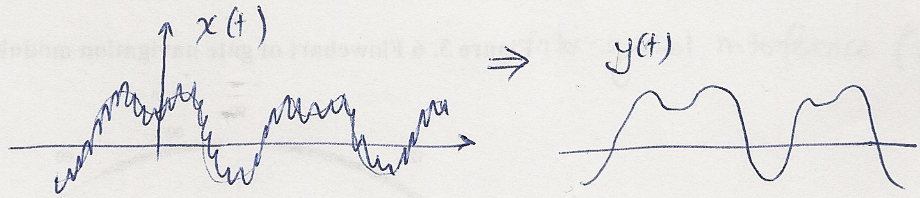
Ex: LPF channel



frequencies below f_{cutoff} remain intact

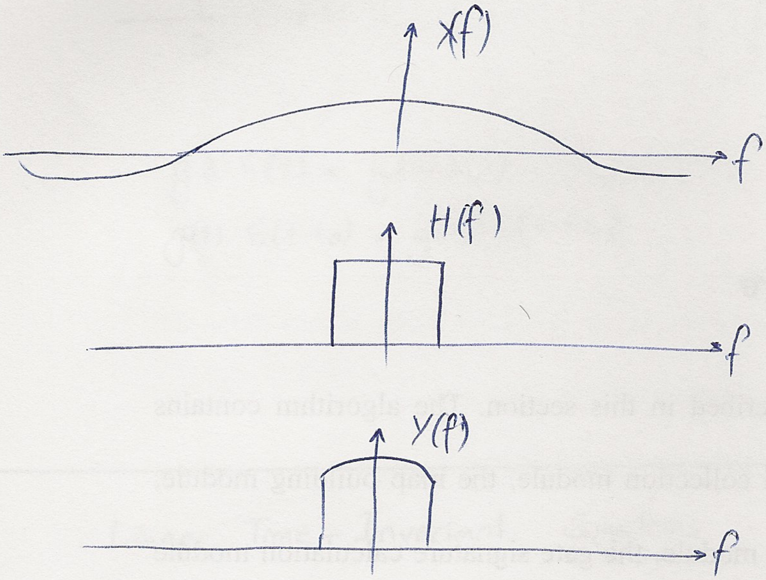
frequencies above f_{cutoff} are suppressed



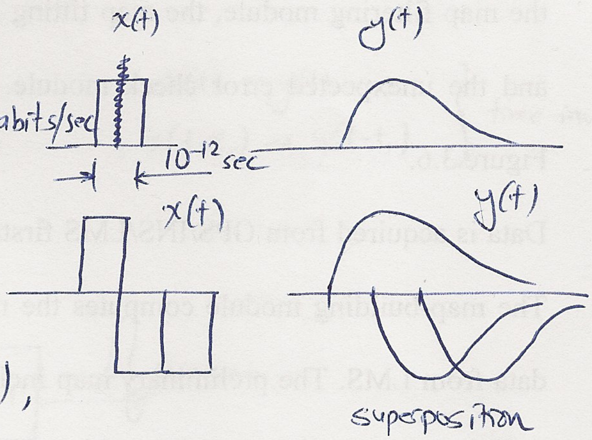


high frequency components have been filtered out.

What happens if we try to pass a very high rate signal from a band-limited channel?



$Y(f) \neq X(f) \rightarrow y(t) \neq x(t)$



We can transmit at any rate (clock speed), but detection will not be possible!

inter-symbol interference (ISI)