CARLETON UNIVERSITY

Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2016

Professor H. Yanikomeroglu **TERM EXAM** **20 October 2016**

Full mark: 100 points – closed-book, one-page aid-sheet and calculators are allowed – 80 mins

Name: **Student #:** **E-mail:**

Q1 [20 pts] – BER Sketching

It is shown in the lectures that the bit error probability (P_e) in an antipodal binary transmission system with a properly designed matched filter receiver in an AWGN channel is given as $P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$. Using the erfc(.) table given in the attached sheet, sketch P_e in the log-log scale. The vertical axis (P_e) ticks should be at 10^{-1}, 10^{-2}, 10^{-3}, and so on, and the horizontal axis (E_b/N_0) ticks should be at -3 dB, 0 dB, 3 dB, 6 dB, and 9 dB.

Q2 [15 pts] – Two-Path Wireless Channel

The impulse response of a two-path wireless channel is given as $h_{ch}(t) = \alpha\, \delta(t) - \alpha\, \delta(t-t_d)$. The distance between the direct path and the reflected path is denoted as d, and the corresponding time difference in arrival times is denoted by t_d. This system uses binary signalling with rectangular pulses and a bit-duration of T seconds.

a) We have two applications with transmission rates R = 100 Kbps (VoIP) and 10 Mbps (high-definition video streaming). For each application, for what d value, we will have $t_d = T$. For each case, comment whether this situation (i.e., $t_d = T$) will occur in indoors or outdoors.

b) In the class, we computed the P_e (probability of error) value for the case when $h_{ch}(t) = \alpha\, \delta(t) + \alpha\, \delta(t-T)$. Compute P_e when $h_{ch}(t) = \alpha\, \delta(t) - \alpha\, \delta(t-T)$. Substantiate your answer with detailed reasoning (omit the effect of the background noise).

Q3 [15 pts] – Noise Power Calculations

a) It is often assumed that the white noise power spectral density is N_0 = -174 dBm/Hz. Show how this value is calculated? [Note that $P_{noise} = kTBF$ Watts, where $k = 1.38\text{x}10^{-23}$ joule/°K]

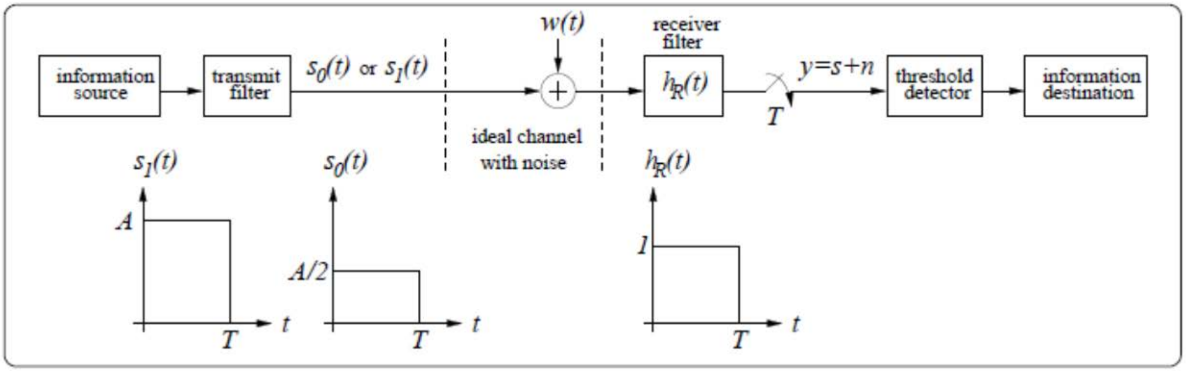
b) The PSD (power spectral density) of filtered noise $n(t)$ is given as

$$S_N(f) = \begin{cases} N_0/2, & -300MHz \le f \le -280MHz \\ N_0/2, & 280MHz \le f \le 300MHz \\ 0, & elsewhere \end{cases} .$$

- Find the bandwidth of $n(t)$.
- Find the total power of $n(t)$ if N_0 = -174 dBm/Hz (assume F = 1 = 0 dB).

Q4 [50 pts] – Binary Signalling with Non-Antipodal Pulses

A baseband binary signalling scheme is used in a zero-mean AWGN channel (with PSD $N_0/2$) as shown in the below figure. Obviously, this is not an "antipodal" signalling scheme since the pulses that represent "1" and "0", $s_1(t)$ and $s_0(t)$, do not have opposite polarity. At the receiver, the decision variable at the output of the sampler is $y = s + n$, where s and n denote the signal noise components, respectively.



(a) (5 pts) Is there ISI in this system (explain)?

(b) (10 pts) Find $s|0$, $s|1$, and n.

(c)

(d) (5 pts) Assuming that 0's and 1's are equally likely, find the threshold level, λ, for the threshold detector.

(e) (10 pts) Find the probability of error, P_e, in term of the erfc(.) function given below:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du.$$

(f) (10 pts) Is the receiver filter given above a matched filter (explain)? Can you design a better receiver filter (i.e., which will yield a smaller P_e)?

(g) (10 pts) Consider a binary NRZ signalling scheme, which is antipodal where a "1" and a "0" are represented by rectangular pulses of amplitude B and $-B$, respectively. Assume that a matched filter is used at the receiver (all other assumptions are the same). For what value of B (found in terms of A), this scheme will have the same P_e as the one described above?