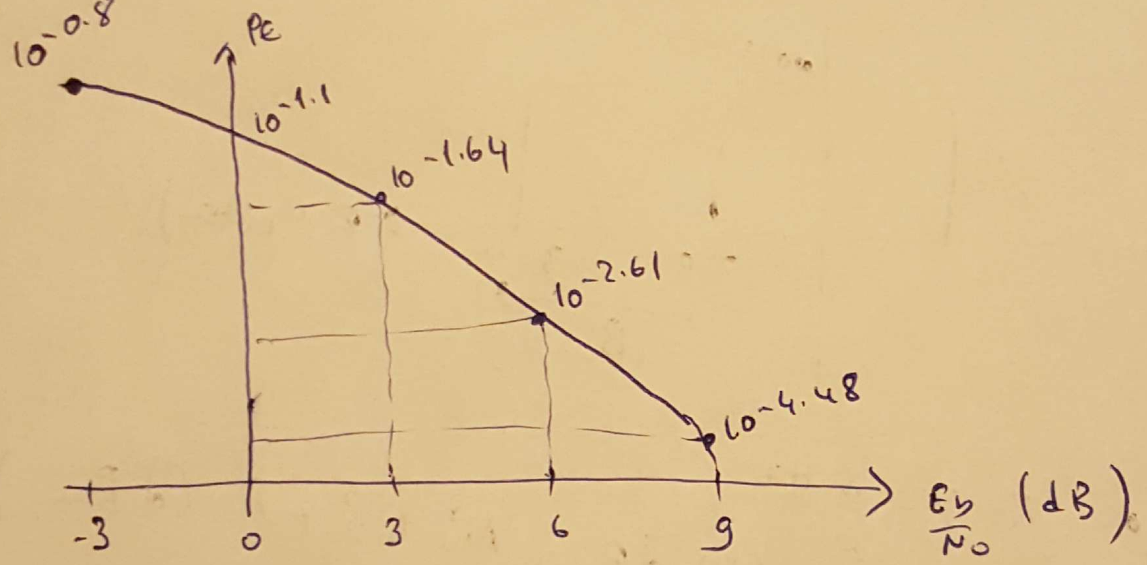
2016 FALL TERM EXAM

Q1.

$\frac{E_b}{N_0}$ (dB) $\in \{-3, 0, 3, 6, 9\}$

$\frac{E_b}{N_0}$ (linear) $\in \{0.501, 1, 1.995, 3.981, 7.943\}$

$P_e \in \{0.1584, 0.0786, 0.0229, 0.0024, 3.3637 \cdot 10^{-5}\}$



Q2. a) $d = cT$, For $R = 100$ Kbps $\rightarrow$ $d = \frac{3 \cdot 10^8}{10^5} = 3000$ m (outdoor)

For $R = 10$ Mbps $\rightarrow$ $d = \frac{3 \cdot 10^8}{10^7} = 30$ m (indoor)

b) $P_e = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 0 = \frac{1}{4}$

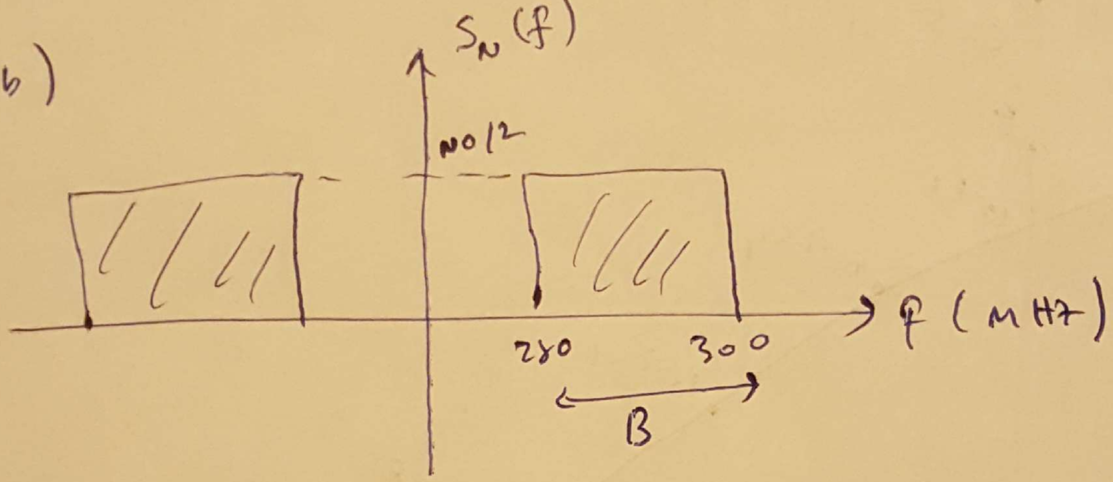
Q3. a) Assuming $T = 15°C$,

$$N_0 = kT = 1.38 \cdot 10^{-23} \cdot (273 + 15)$$

$$= -204 \text{ dB W/Hz}$$

$$= -174 \text{ dBm/Hz}$$

b)



- $B = 20$ MHz
- $P_N = \int_{-\infty}^{\infty} S_N(f) \cdot df = N_0 + B + F$ (dBm)

$$= -174 + 73 = -101 \text{ dBm}$$

Q4.

a) $s_i(t)$: time-limited

chanel : no shaping

$\Big\}$ not broadened → no ISI

b) $s|0 = \frac{AT}{2} \quad \left(s_0(t) * h_R(t) \Big|_{t=T} = \frac{AT}{2} \right)$

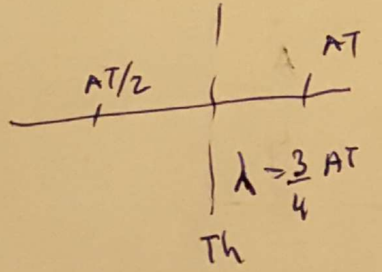
$s|1 = AT$

$n \sim N\left(0, \frac{N_0 T}{2}\right) . \quad \left(w(t) * h_R(t) \Big|_{t=T} \right)$

$$\sigma_n^2 = E[n^2] - E^2[n] = \int_{-\infty}^{\infty} S_N(f) \cdot df = \frac{N_0}{2} \int_{-\infty}^{\infty} H_R(f) \, df$$

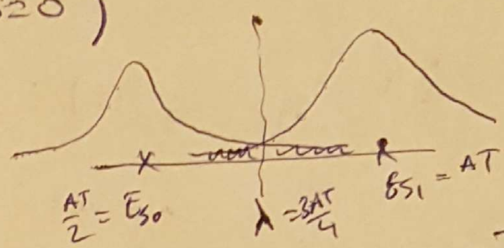
$$= \frac{N_0}{2} \int h^2(t) \cdot dt = \frac{N_0 T}{2}$$

d) $\lambda = \dfrac{AT + \frac{AT}{2}}{2} = \dfrac{3AT}{4}$



e) $P_e = \frac{1}{2} Pr(y=0 \mid s=1) + \frac{1}{2} Pr(y=1 \mid s=0)$

$$\left\{ \begin{array}{l} s=1, \text{ then } y \sim N\left(AT, \frac{N_0 T}{2}\right) \\ s=0, \text{ then } y \sim N\left(\frac{AT}{2}, \frac{N_0 T}{2}\right) \end{array} \right\}$$



$$P_e = \frac{1}{2} Pr\left\{ N\left(\frac{AT}{2}, \frac{N_0 T}{2}\right) > \frac{3AT}{4} \right\} + \frac{1}{2} Pr\left\{ N\left(AT, \frac{N_0 T}{2}\right) < \frac{3AT}{4} \right\}$$

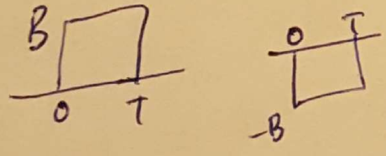
Note that $Pr\{X > x\} = Q\left(\frac{x-\mu}{\sigma}\right)$ where $X \sim N(\mu, \sigma^2)$

$$= Q\left(\frac{A\sqrt{2T}}{4\sqrt{N_0}}\right) = \boxed{\frac{1}{2} \cdot \text{erfc}\left(\frac{A\sqrt{T}}{4\sqrt{N_0}}\right)}$$

f) Matched Filter since $\boxed{h_{Rx}(t) = h_{Tx}(T-t)}$

s_0, s_1 : one pulse PAM

g)



$$\frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{B^2 T}{N_0}}\right)$$

$$\frac{B^2 T}{N_0} = \frac{A^2 T}{16 N_0} \Rightarrow \boxed{B = \frac{A}{4}}$$