CARLETON UNIVERSITY

Department of Systems and Computer Engineering

SYSC 4600 **Digital Communications** **Fall 2016**

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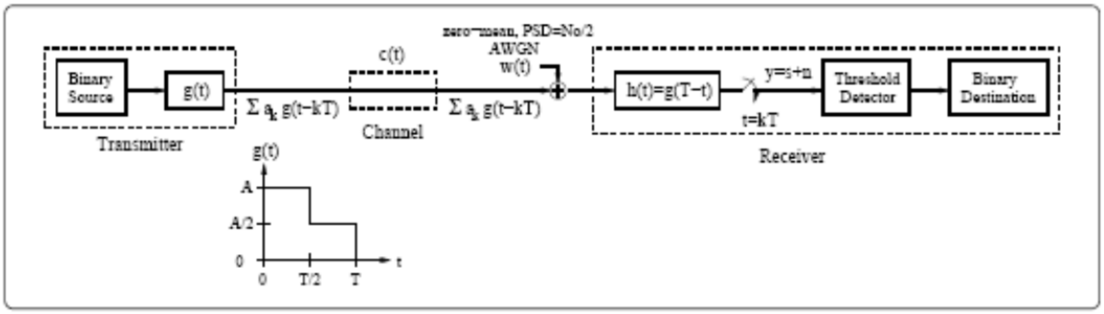
Due on : **Will not be collected (for studying purposes only)**

Question 2 (60 points) **Bit Error Rate (BER) Calculation**



A baseband binary signalling scheme is shown in the above figure.

- The information bits are equally-likely.
- Signalling is antipodal: $a_k = \{-1, 1\}$.
- The source produces one bit every T seconds.
- The channel is ideal with AWGN.
- The receiver is a matched filter with $h(t) = g(T - t)$.

(a) (5 pts) Give the expression for $c(t)$, the channel impulse response. Show mathematically that $(\sum a_k g(t - kT)) * c(t) = \sum a_k g(t - kT)$, where * denotes convolution.

(b) (5 pts) Find E_b, the received bit energy, in terms of A and T.

(c) (5 pts) Sketch $h(t) = g(T - t)$.

(d) (5 pts) The decision variable at the output of the sampler has a deterministic component s due to the signal and a random component n due to noise; that is $y = s + n$. Find s given that a "1" is transmitted.

(e) (10 pts) Find the expression for n in the form of an integral equation (simplify as much as you can). [Hint: n can be written as the sum of two integrals.]

(f) (10 pts) Find the mean and variance of n in terms of A, T, and N_o. Show that $\sigma_n^2 = E[n^2]$ can be written as $E_b N_o/2$. [Hint: in the evaluation of $E[n^2]$, the cross terms will disappear − why?]

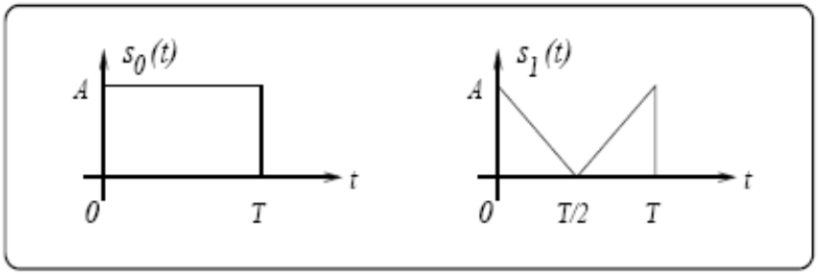
(g) (5 pts) Find $f_Y(y|0)$ and $f_Y(y|1)$.

(h) (5 pts) Sketch $f_Y(y|0)$ and $f_Y(y|1)$ together. Indicate the decision threshold.

(i) (10 pts) The probability of error is $P_e = \frac{1}{2}P(0|1) + \frac{1}{2}P(1|0)$, and due to symmetry it reduces to $P_e = P(1|0)$. Find P_e in terms of the erfc function (show the intermediate steps). [Hint: erfc $(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-z^2} dz$]

Question 3 (44 points) **Signal Space Representation**

Two signals, $s_0(t)$ and $s_1(t)$, are shown in the below figure.



(a) (4 pts) Find the energy of $s_0(t)$, E_{s_0}, and that of $s_1(t)$, E_{s_1}. Show that $E_{s_0}/E_{s_1} = 3$.

(b) (3 pts) Find the inner product of $s_0(t)$ and $s_1(t)$; i.e., $(s_0(t), s_1(t))$.

(c) (5 pts) Show $s_0(t)$ and $s_1(t)$ as vectors, indicate the angle between those two vectors. [*Hint:* this part can be solved without using the Gram-Schmidt orthogonolization method.]

(d) (4 pts) Draw an $s_2(t)$ which is orthogonal to both $s_0(t)$ and $s_1(t)$.
[*Hint:* this part can be solved by inspection.]

Suppose that $s_0(t)$ and $s_1(t)$ represent the transmitted signals for "0" and "1", respectively, in a baseband binary communications system. Assume that 0's and 1's are equally-likely. Assume further that the channel is ideal and the noise is AWGN with PSD $= N_o/2$.

(e) (3 pts) If $T = 2.5$ microseconds (refer to the above figure), find the bit rate. How long will it take to transfer a file of 5 Megabytes with this signalling scheme?

(f) (7 pts) Draw the optimum receiver for this system using one single matched filter. Draw the impulse response of that filter.

(g) (5 pts) Find the probability of error in terms of E_{av} (average bit energy).
[*Hint:* this part can be solved by inspection.]

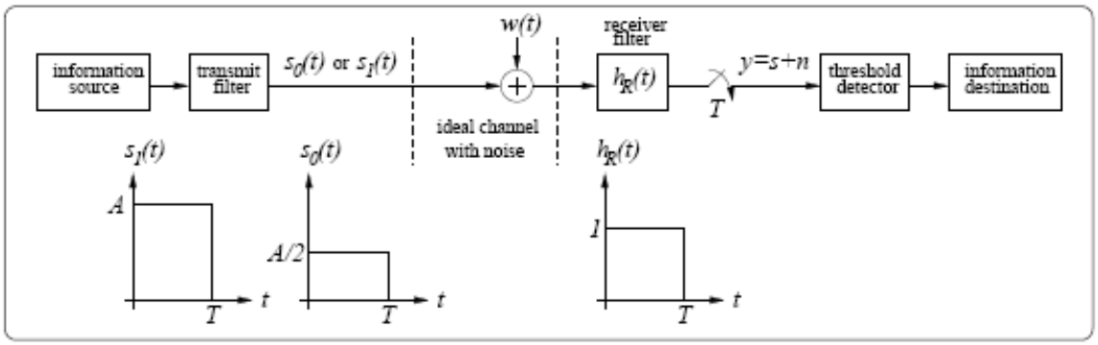
(h) (4 pts) Suppose that it is given that $s_1(t)$ has to be used to represent "1", and you have flexibility in choosing the signal representing "0"; let us denote this signal by $s_0'(t)$. Determine $s_0'(t)$ that will yield the minimal probability of error at the receiver with the following constraint: the energy of $s_0'(t)$ must be equal to that of $s_0(t)$, in order to have a fair comparison (in other words, E_{av} is preserved). Assume that the receiver is optimal.

(i) (4 pts) Give the probability of error expression, in terms of E_{av}, for the signalling scheme described in part (h). [*Hint:* this part can be solved by inspection.]

(j) (5 pts) Does the signalling scheme composed of $s_0'(t)$ and $s_1(t)$ have DC power? Explain. In this context, is this signalling scheme suitable for dial-up modems which use twisted-pair telephone channels? Explain.

Question 2 (80 pts) **Binary Signalling with Non-Antipodal Pulses**

A baseband binary signalling scheme is used in a zero-mean AWGN channel (with PSD $N_0/2$) as shown in the below figure. Obviously, this is not an "antipodal" signalling scheme since the pulses that represent "1" and "0", $s_1(t)$ and $s_0(t)$, do not have opposite polarity. At the receiver, the decision variable at the output of the sampler is $y = s + n$, where s and n denote the signal noise components, respectively.



(a) (5 pts) Is there ISI in this system (explain)?

(b) (10 pts) Find $s|0$, $s|1$, and n.

(c) (10 pts) Find $f_Y(y|0)$ and $f_Y(y|1)$ and sketch them together.

(d) (5 pts) Assuming that 0's and 1's are equally likely, find the threshold level, λ, for the threshold detector.

(e) (10 pts) Find the probability of error, P_e, in term of the erfc(.) function given below:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du.$$

(f) (10 pts) Is the receiver filter given above a matched filter (explain)? Can you design a better receiver filter (i.e., which will yield a smaller P_e)?

(g) (10 pts) Consider a binary NRZ signalling scheme, which is antipodal where a "1" and a "0" are represented by rectangular pulses of amplitude B and $-B$, respectively. Assume that a matched filter is used at the receiver (all other assumptions are the same). For what value of B (found in terms of A), this scheme will have the same P_e as the one described above?

(h) (5 pts) What is the average instantaneous power, P_{av}, for the above described non-antipodal and antipodal cases? ($P_{av} = \frac{1}{2}P_{s_0} + \frac{1}{2}P_{s_1}$, where P_{s_0} and P_{s_1} denote the instantaneous powers for the pulses that represent "0" and "1", respectively.)

(i) (5 pts) For a given P_{av}, is an antipodal or a non-antipodal scheme yields less probability of error?

(j) (10 pts) If the $s_0(t)$ and $s_1(t)$ shown in the above figure were centred around $t = 0$ (that is, the pulses are defined in the interval $t \in [-T/2, T/2]$), what would have been the sampling time at the receiver? Sketch the impulse response of the corresponding receiver filter.