

**CARLETON UNIVERSITY**  
**Department of Systems and Computer Engineering**

**SYSC 4600**

**Digital Communications**

**Fall 2016**

**Assignment 1** – 130 points

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Due on : **Will not be collected (for studying purposes only)**

**Q1. (10 pts)** There is a linear time-invariant (LTI) channel with impulse response  $h(t)$ . The input and output of the channel are denoted by  $x(t)$  and  $y(t)$ , respectively. The corresponding Fourier Transforms are  $H(f)$ ,  $X(f)$ , and  $Y(f)$ , respectively. Write the output in terms of the input and channel

- in time domain,
- in frequency domain.

**Q2. (10 pts)** Sketch the power spectral density (PSD) of white noise.

**Q3. (10 pts)** Sketch the PDF (probability density function) and CDF (cumulative distribution function) curves for a Gaussian random variable.

**Q4. (10 pts)** If  $\text{SNR} = 20$  in the linear domain, what is SNR in dB?

**Q5. (10 pts)** In a statistical experiment, the five possible outcomes for the random variable  $X$  are as follows:  $\{1, 4, -3, -2, 5\}$ . All the outcomes are equally-likely with probability  $1/5$ . Find the mean and variance of  $X$ .

**Q6. (10 pts)** Sketch  $\cos(2\pi f_c t + \pi/4)$ .

**Q7. (20 pts)**  $X$  is uniform random variable taking values in the range  $[0, 4]$ .

# Draw the PDF of  $X$ .

# Draw the CDF of  $X$ .

# Find the variance of  $X$ .

# Find  $\text{Prob}(2 < X < 3)$ .

# Find  $\text{Prob}(X=3)$ .

**Q8. (20 pts)** Consider a wireless channel modelled as an LTI (linear, time-invariant) system with an impulse response  $h(t) = a\delta(t) + a\delta(t-T/2)$ , where  $a$  is a constant. Let us assume that  $x(t)$  is the transmitted signal representing one single bit; it is a rectangular function with amplitude 1 and duration  $[0, T]$ . Find the received signal  $y(t)$ .

**Q9. (10 pts)** We have two signals  $x(t)$  and  $y(t)$ ; the former one is a baseband signal and the latter a bandpass signal. The Fourier transforms of these signals,  $X(f)$  and  $Y(f)$ , are given as follows:

$$X(f) = \begin{cases} \alpha, & -3\text{MHz} \leq f \leq 3\text{MHz} \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad Y(f) = \begin{cases} \beta, & -200\text{MHz} \leq f \leq -150\text{MHz} \\ \beta, & 150\text{MHz} \leq f \leq 200\text{MHz} \\ 0, & \text{elsewhere} \end{cases}.$$

Find the bandwidth of these two signals.

**Q10. (20 pts)** The probability of error (bit error rate) versus SNR curve for BPSK (binary phase shift keying) is given below. If 1 million bits are received at 8 dB SNR, on average how many of them will be detected erroneously?

