CARLETON UNIVERSITY
Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2015

Professor H. Yanikomeroglu **21 October 2015**

Full mark: 100 points – closed-book, two-page aid-sheet and calculators are allowed – 80 mins

Name: **Student #:** **E-mail:**

Q1 (20 pts) – BER Calculation in a 2-Path Wireless Channel

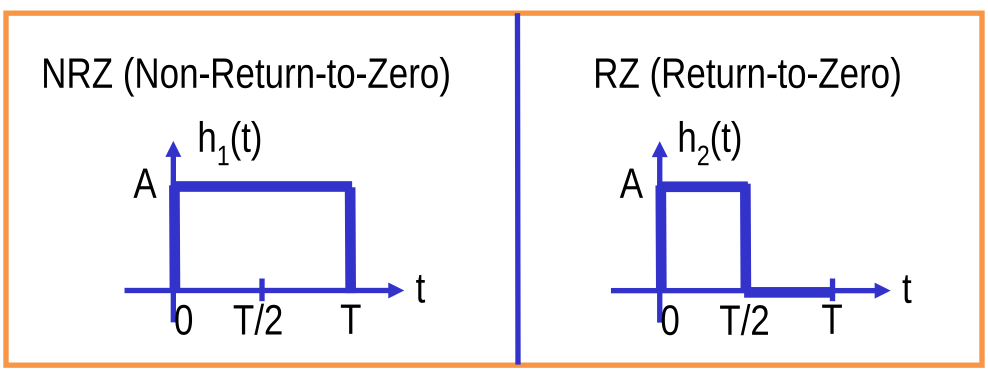
A large file composed of 0`s and 1`s is to be transmitted through a wireless channel. Binary 1 is represented by the rectangular function $x(t)$ with amplitude A and duration [0, T]; binary 0 is represented by $-x(t)$.

Consider a wireless channel modelled as an LTI (linear, time-invariant) system with an impulse response $h(t) = a\delta(t) + a\delta(t-3T)$, where a is a constant and T is the bit duration.

Assume that there is no background noise. Find the probability of bit error at the output of the receiver detector.

Q2 (20 pts) – Power Spectral Density (PSD)

Please refer to the below figure.



In polar NRZ signalling, $h_1(t)$ and $-h_1(t)$ are used to represent the message bits 1 and 0, respectively.
In polar RZ signalling, $h_2(t)$ and $-h_2(t)$ are used to represent the message bits 1 and 0, respectively.

The PSDs for the two signalling schemes are given as $S_{NRZ}(f)=|H_1(f)|^2/T$ and $S_{RZ}(f)=|H_2(f)|^2/T$, where T is the bit duration (=1/R, where R is the bit-rate).

a) Find the $S_{RZ}(f)$ expression.
b) Sketch $S_{RZ}(f)$.
c) What is the bandwidth of the RZ signalling in terms of R? (State what BW definition you used.)

Q3 (20 pts) – Short Questions

a) **Impact of Temperature on SNR:** The noise power is given as $P_{noise} = kTBF$, where

> k: Boltzmann's constant = 1.38×10^{-23} J/K
> T: Temperature in degrees Kelvin (^{0}K = 273 + ^{0}C)
> B: Bandwidth
> F: Noise figure = 4 dB

The SNR value at 15^0C is given as 7 dB.

- What will be the SNR value if the temperature increases from 15^0C to 25^0C while everything else remains the same?
- What will be the SNR value at 15^0C, if the noise figure increases from 4 dB to 6 dB while everything else remains the same?

b) **AWGN Channel:** Write the channel impulse response expression, $h_c(t)$, for an AWGN channel.

Q4 (20 pts) – Probability of Error Sketching

In a binary antipodal signalling scheme with an AWGN channel and a matched filter receiver, the expression for the probability of error, P_e, was driven in the class as $P_e = \frac{1}{2} erfc\left(\sqrt{SNR}\right)$.

When SNR is sufficiently high, P_e can be approximated as $P_e = \frac{1}{2} erfc\left(\sqrt{SNR}\right) \cong \frac{1}{2\sqrt{\pi}} \frac{e^{-SNR}}{\sqrt{SNR}}$.

Sketch P_e in the log-log scale for the SNR range [5dB 10dB].

Q5 (20 pts) – Amplification at the Receiver

Consider the matched filter receiver discussed in the lectures. Let y be the decision variable at the input of the threshold detector (i.e., right after the matched filter). As we know, $y = s + n$, where s is the signal component and n is the noise component. It was shown that $s = \pm E_b$ and $n : G(0;\sigma^2 = E_b N_0 / 2)$. SNR at that point (at the input of the threshold detector) was computed as $SNR_Y = s^2/E[n^2] = 2E_b/N_0$, where E_b is the bit energy, N_0 is the noise PSD, and E is the expectation operator.

Now, an amplifier with an amplification factor of B (value in linear scale) is inserted between the matched filter and the threshold detector. In this revised system, let z be the decision variable at the input of the threshold detector; i.e., $z = By$.

- Find the SNR expression at the output of the amplifier (SNR_Z) and compare that with the SNR value at the input of the amplifier (SNR_Y).
- What will the impact of the amplifier be on the reliability (BER); discuss qualitatively.