CARLETON UNIVERSITY
Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2014

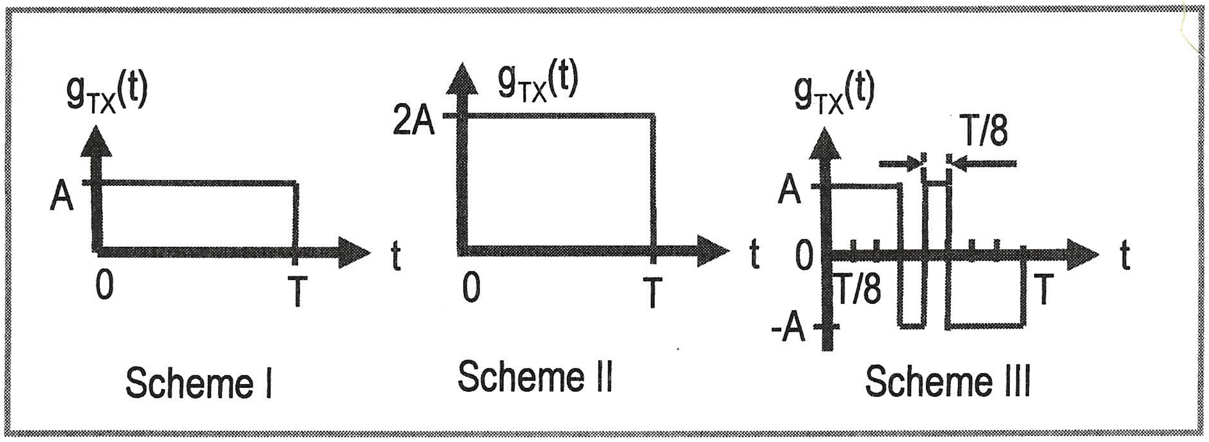
Professor H. Yanikomeroglu **22 October 2014**

Full mark: 160 points – closed-book, one-page aid-sheet and calculators are allowed – 80 mins

Name: **Student #:** **E-mail:**

Question 1 (40 pts) – Short Questions

(a) **[15 pts]** In a binary antipodal digital communications system the transmitted signal can be shown as $x(t) = \Sigma\, a_k\, g_{TX}(t-kT)$, where a_k is -1 or 1, and $g_{TX}(t)$ denotes the pulse shaping filter at the transmitter. There are three different possibilities for $g_{TX}(t)$ as shown below:



It is given that the bandwidth of Scheme I is 3.2 MHz.

- Find the bandwidth of Scheme II.
- Find the bandwidth of Scheme III.

$\boxed{\text{5 pts}}$ * Since there is no change in pulse width ∴ BW = 3.2 MHz

$\boxed{\text{10 pts}}$ * BW of scheme III = 8 X 3.2 = 25.6 MHz

b) [10 pts] In a receiver, noise power is given as P_N = -99 dBm. If N_0 = -174 dBm/Hz, and the noise figure as 8 dB. Find the signal BW.

$P_N = N_0 * B * F$ (linear domain)

$P_N = -99\, dBm = -129\, dBW = 10^{-12.9}$ watt

$N_0 = -204\, dB/Hz = 10^{-20.4}$ watt/Hz

$F = 10^{0.8}$

$B = \frac{P_N}{N_0 F} = 10^{6.7} = 31.6$ MHz

c) [15 pts] Suppose that you are involved in the design of a next-generation WLAN standard, say 802.11hy, that operates in the millimeter wave. The target downlink peak rate is 40 Gbps. Choose some appropriate

- bandwidth,
- peak spectral efficiency, and
- number of antennas (for MIMO gain)

values for this system.

BW = 1 GHz

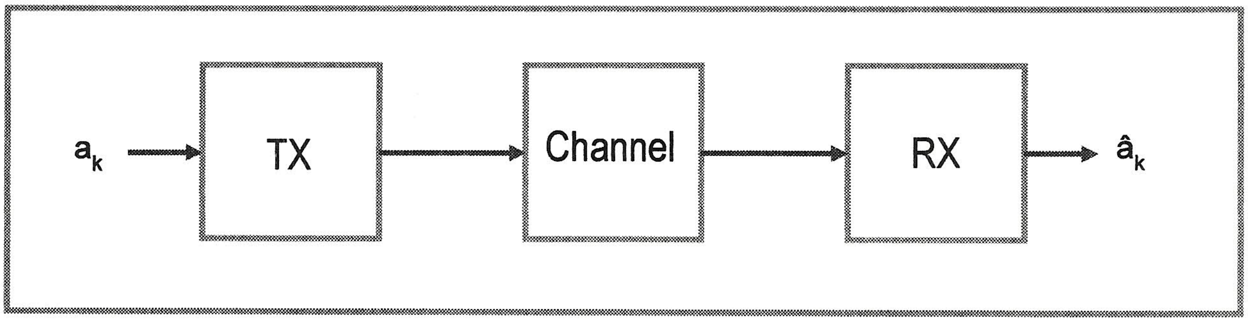
Peak spectral efficiency = 6 b/s/Hz

Number of antennas = 8

Peak rate = 1 GHz * 6 b/s/Hz x 8 = 48 Gb/s > 40 Gb/s

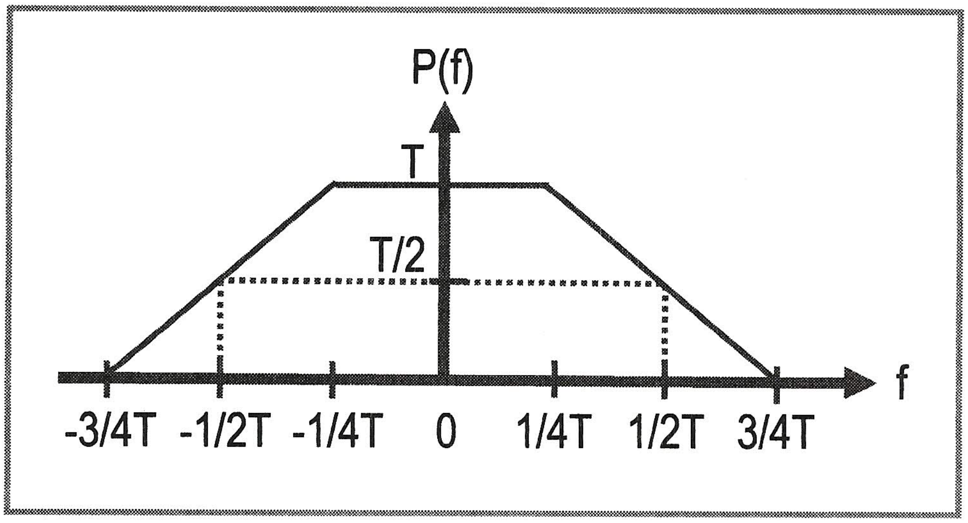
Question 2 (40 pts) – Intersymbol Interference (ISI)

Consider the digital communications system below. Since the major concern is ISI, the background noise is omitted.



$P(f) = |H_{TX}(f)|\ |H_{CH}(f)|\ |H_{RX}(f)|$. As discussed in lectures, if $P(f)$ has the following property ("Nyquist no-ISI condition"), $\Sigma\ P(f\text{-}k/T) = T$, then there in no ISI in the system (T denotes symbol time).

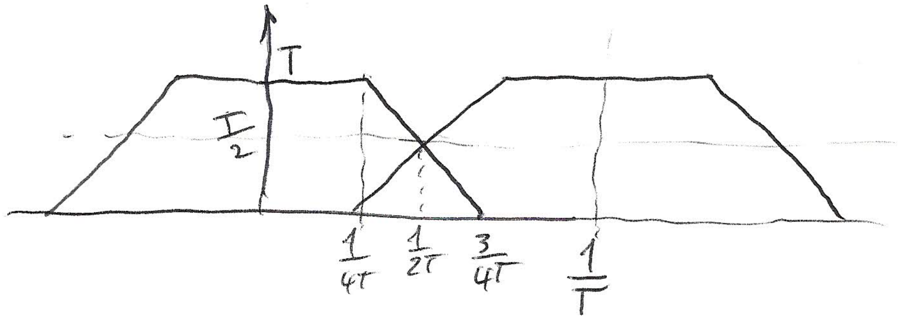
a) Next, consider the following $P(f)$.



Does this system satisfy the Nyquist no-ISI criterion?

Yes

10 pts



b) If the TX and RX filters are designed as matched filter pairs under the assumption that the channel is perfect (i.e., $h_{CH}(t) = \delta(t)$), sketch $|H_{TX}(f)|$ and $|H_{RX}(f)|$.

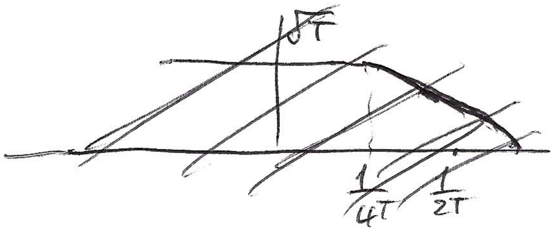
$\boxed{15 \text{ pts}}$

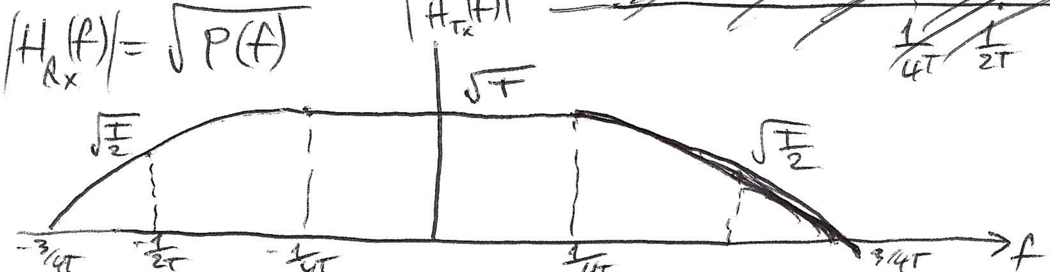
$$P(f) = |H_{Tx}(f)|\,|H_{Rx}(f)|\,|H_{CH}(f)|$$

$$|H_{CH}(f)| = 1$$

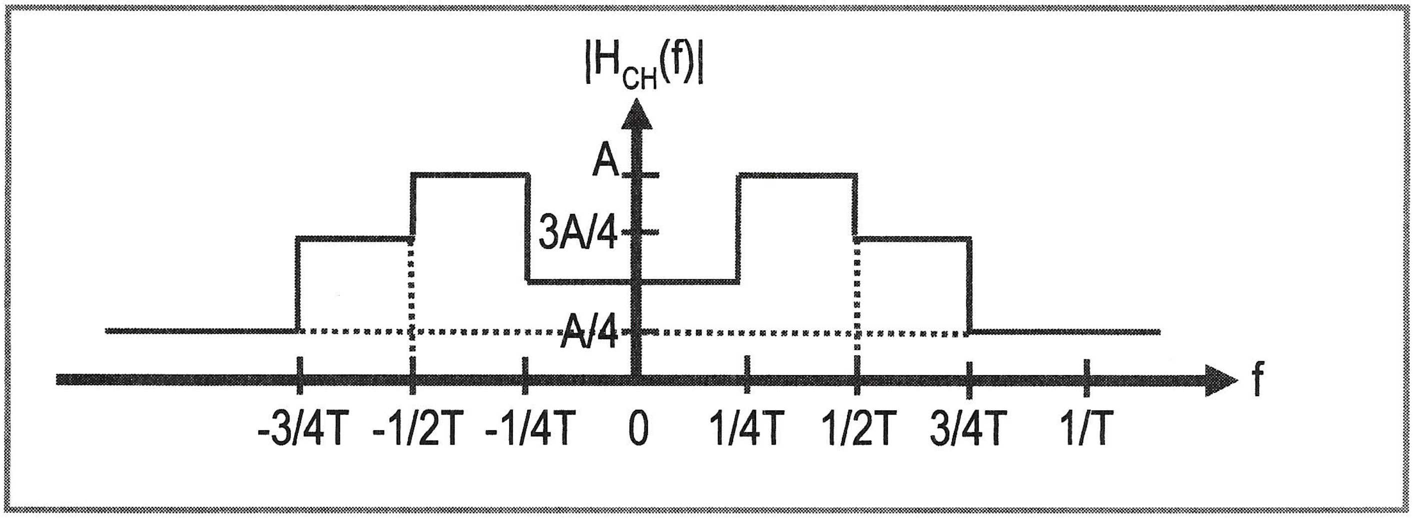
Since $|H_{Tx}(f)| = |H_{Rx}(f)|$

$$\therefore |H_{TX}(f)| = |H_{Rx}(f)| = \sqrt{P(f)}$$





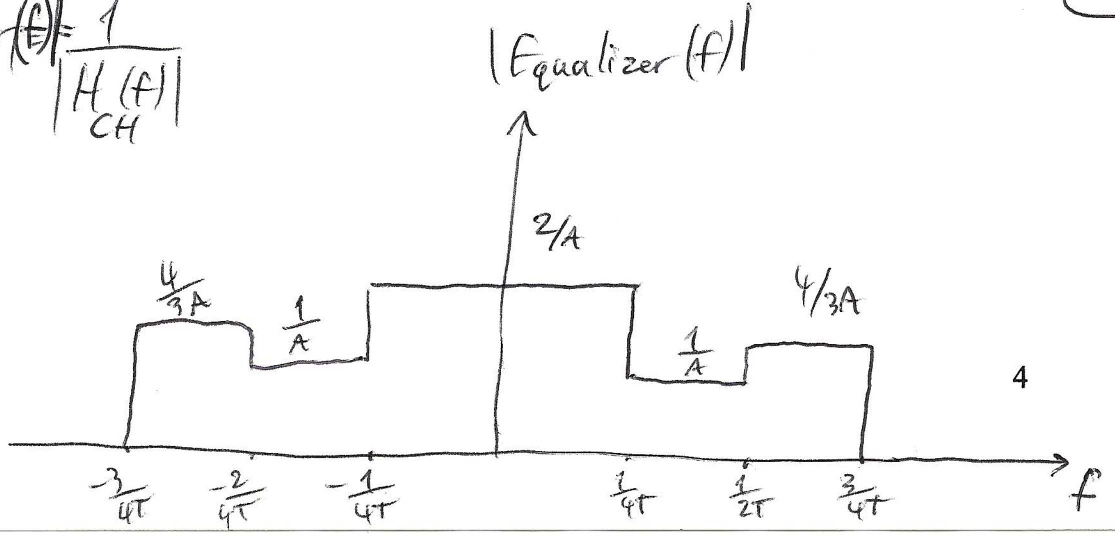
c) Now, in a system with the matched TX and RX filters as obtained in part b), the following channel frequency response measurement is made:



Sketch the equalizer transfer function (magnitude) which will completely remove the effect of the channel yielding a no-ISI situation.

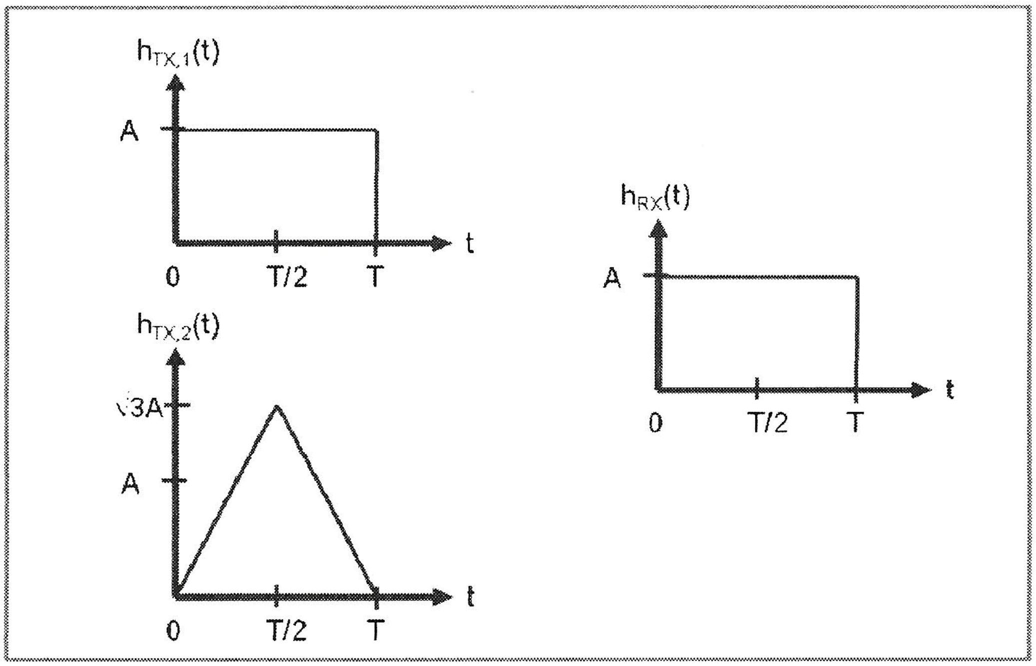
$\boxed{15 \text{ pts}}$

$$|Equalizer(f)| = \frac{1}{|H_{CH}(f)|}$$



Question 3 (40 pts) – Probability of Error Analysis

Consider a binary antipodal signalling scheme with AWGN (power spectral density = $N_0/2$), in the presence of a perfect channel, $h_{CH}(t) = \delta(t)$. The information bits are equally-likely; the transmission rate is $R = 1/T$ bits/sec.



The transmitter pulse shaping filter impulse responses are given above for two different schemes, along with the receiver filter impulse response which is the same for both schemes. Clearly, the receiver is not matched to the transmitter in Scheme 2.

The receiver filter is followed by a sampler (which samples at every T sec), which is followed by a threshold detector with a threshold value of 0.

We will compare the performance of the two signalling schemes. The probability of bit error is computed in the class for Scheme 1:

$$P_{e,1} = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_{b,1}}{N_0}}\right).$$

a) Show that $E_{b,2} = E_{b,1} = A^2 T$. Let us denote this common bit energy value as E_b.

b) Staring from the first principles, derive the $P_{e,2}$ (error probability for Scheme 2). You can get a generous partial mark if you solve the question by inspection.

c) In order for both schemes to have the same P_e value (i.e., $P_{e1}=P_{e2}$), by how many dBs the transmit power of Scheme 2 should be increased or decreased?

Note: $erfc(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-z^2} dz$.

$\boxed{10\,pts}$

ⓐ $E_{bav} = P_0 E_{b_0} + P_1 E_{b1}$

$= 0.5\, E_{b_0} + 0.5\, E_{b_1}$

$$E_{b_0} = E_{b_1} = \int_{-\infty}^{\infty} |h(t)|^2 dt = \int_0^{\frac{T}{2}} \frac{3A^2}{T^2/4} t^2 dt + \int_{\frac{T}{2}}^{T} \left(\frac{-\sqrt{3}A}{\frac{T}{2}} t + 2\sqrt{3}A\right)^2 dt$$

$$= \frac{3A^2}{T^2/4}\left[\frac{t^3}{3}\right]_0^{\frac{T}{2}} + \left[\frac{3A^2}{T^2/4}\frac{t^3}{3} - \frac{24A^2}{T}\frac{t^2}{2} + 12A^2 t\right]_{\frac{T}{2}}^{T}$$

$$= \frac{A^2T}{2} + \frac{4A^2}{T^2}\left[T^3 - \frac{T^3}{8}\right] - \frac{12A^2}{T}\left[T^2 - \frac{T^2}{4}\right] + 6A^2T$$

$$= \frac{A^2T}{2} + \frac{7A^2T}{2} - 9A^2T + 6A^2T = A^2T$$

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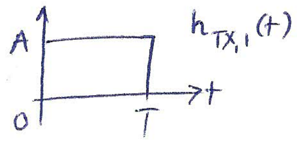
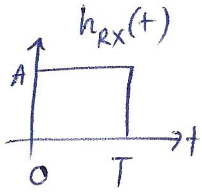
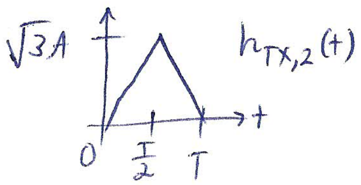
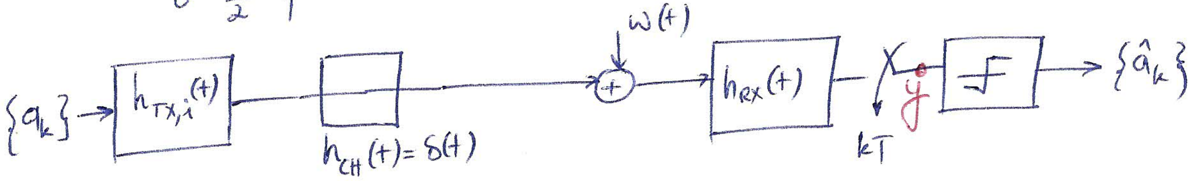
Q3)





$E_{b,2} = E_{b,1} = A^2T : E_b$





$$P_{e,1} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{A^2T}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}$$

25pts + 10 bonus pts

b) $P_{e,2} = P_1 P_{e|1} + P_{-1} P_{e|-1} = \frac{1}{2}\left(P_{e|1} + P_{e|-1}\right)$

$P_{e|1} = P(\hat{a} = -1 \mid a = 1) = P(y < 0 \mid a = 1)$

$$y = \left(a\, h_{TX,2}(t) + w(t)\right) * h_{Rx}(t)\Big|_{t=T} = a\, h_{TX,2}(t) * h_{Rx}(t)\Big|_{t=T} + \underbrace{\overbrace{w(t) * h_{Rx}(t)}^{n(t)}\Big|_{t=T}}_{n}$$

$$y = \begin{cases} \frac{\sqrt{3}}{2}E_b + n, & a = 1 \\ -\frac{\sqrt{3}}{2}E_b + n, & a = -1 \end{cases}$$

$$= a\,\frac{\sqrt{3}A^2T}{2} + n \;\; \hookrightarrow G(0; \sigma_n^2)$$

$$\sigma_n^2 = E[n^2] - \cancel{E^2[n]}_{0} = \text{Power}_n = \int_{-\infty}^{\infty} S_n(f)\,df$$

$$= \int_{-\infty}^{\infty} S_w(f)\,|H_{Rx}(f)|^2 df = \frac{N_0}{2}\int_{-\infty}^{\infty} |H_{Rx}(f)|^2 df$$

$$= \frac{N_0}{2}E_b$$

$P_{e|1}$

$= P(y < 0 \mid a = 1)$

$= P\left(\frac{\sqrt{3}}{2}E_b + n < 0\right)$

$= P\left(n < -\frac{\sqrt{3}}{2}E_b\right) = P\left(n > \frac{\sqrt{3}}{2}E_b\right)$

$$= \int_{\frac{\sqrt{3}}{2}E_b}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-\frac{n^2}{2\sigma_n^2}}\, dn$$

$\frac{n^2}{2\sigma_n^2} = z^2 \rightarrow dn = \sqrt{2}\,\sigma_n\, dz$

$n = \frac{\sqrt{3}}{2}E_b \rightarrow z = \frac{n}{\sqrt{2}\sigma_n} = \frac{\sqrt{3}}{2}E_b\frac{1}{\sqrt{2}\sigma_n}$

$$= \int_{\frac{\sqrt{3}}{2\sqrt{2}}\frac{E_b}{\sigma_n}}^{\infty} \frac{1}{\cancel{\sqrt{2}}\sqrt{\pi}\,\cancel{\sigma_n}}\, e^{-z^2}\, \cancel{\sqrt{2}}\,\cancel{\sigma_n}\, dz$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{3}}{2\sqrt{2}}\frac{E_b}{\sigma_n}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{3}}{2\cancel{\sqrt{2}}}\frac{E_b}{\sqrt{\frac{N_0}{\cancel{2}}}\sqrt{E_b}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{3}}{2}\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{3}{4}\frac{E_b}{N_0}}$$

due to symmetry

$P_{e|-1} = P_{e|1}$

$\rightarrow P_{e,2} = P_{e|1} = \boxed{\frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{3}{4}\frac{E_b}{N_0}}}$

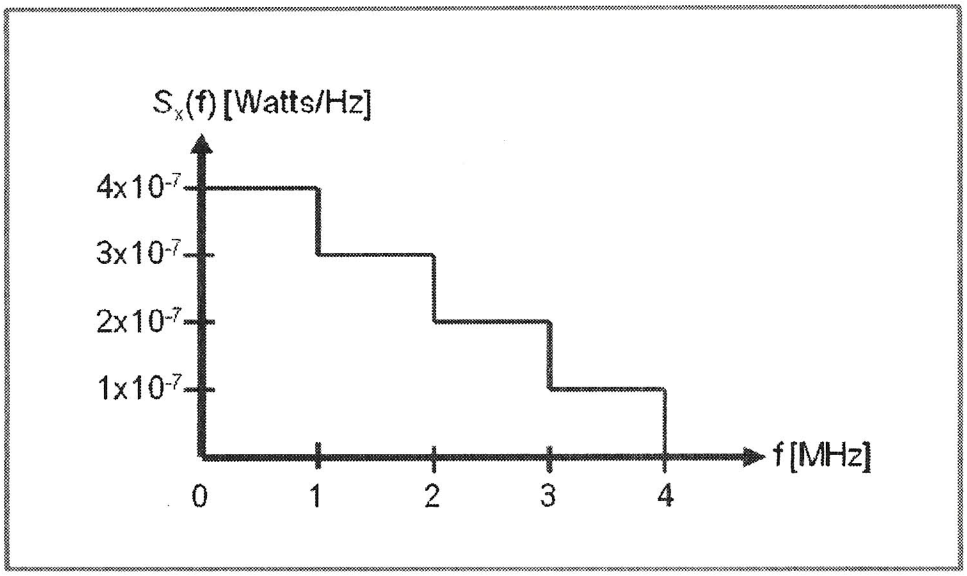
c) $E_{b,2} = \frac{4}{3}E_{b,1} \rightarrow P_{e,2} = P_{e,1}$

increase TX power by $10\log_{10}\frac{4}{3}$ dB $= 1.25$ dB

5pts

Question 4 (40 pts) – Power Spectral Density

The power spectral density, $S_x(f)$, for a digital signalling scheme is given below:



a) Find the total power of this signalling scheme.

b) How much power does this signalling scheme has between 2 MHz and 3 MHz?

c) How much power does this signalling scheme has at 3 MHz?

d) Find the absolute bandwidth of this signalling scheme.

e) $BW_{90\%}$ (90%-bandwidth) is defined as the frequency below which 90% of the total power is confined to. Find $BW_{90\%}$ for this signalling scheme.

(a) $P_{total} = \int_{-\infty}^{\infty} S_x(f)\,df = 2\int_{0}^{\infty} S_x(f)\,df = 2\times10\times10^{-7}\times10^{6} = 2\text{ watts}$

(b) $P_{2\to3MHz} = 2\int_{2M}^{3M} S_x(f)\,df = 2\times10^{-7}\times(3M-2M) = 0.4\text{ watts}$

(c) $P_{3MHz} = 2\int_{3M}^{3M} S_x(f)\,df = 0$

(d) B.W = 4 MHz

(e) $P_{0\to3MHz} = 2\int_{0}^{3M} S_x(f)\,df = 1.8\text{ watt} = 0.9\,P_{total}$

$\therefore BW_{90\%} = 3MHz$