CARLETON UNIVERSITY

Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2012

Professor H. Yanikomeroglu **18 October 2012**

Full mark: 115 points – closed-book, one-page aid-sheet and calculators are allowed – 80 mins

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Question 1 (40 pts) – Short Questions

- **(5 pts)** State the two major impairments in a digital communications system.

- **(5 pts)** Explain what "impulse response" means.

- **(5 pts)** Write the impulse response of an ideal channel.

- **(5 pts)** Sketch the power spectral density (PSD) of white noise.

- **(10 pts)** There is a linear time-invariant (LTI) channel with impulse response *h(t)*. The input and output of the channel are denoted by *x(t)* and *y(t)*, respectively. Write the output in terms of the input and channel
  - in time domain,
  - in frequency domain.

  If the input is a random process with PSD $S_X(f)$, write the output PSD, $S_Y(f)$.

- **(10 pts)** We have two signals *x(t)* and *y(t)*; the former one is a baseband signal and the latter a bandpass signal. The Fourier transforms of these signals, *X(f)* and *Y(f)*, are given as follows:

$$X(f)=\begin{cases}\alpha, & -3MHz \le f \le 3MHz \\ 0, & elsewhere\end{cases} \quad \text{and} \quad Y(f)=\begin{cases}\beta, & -200MHz \le f \le -150MHz \\ \beta, & 150MHz \le f \le 200MHz \\ 0, & elsewhere\end{cases}.$$

  Find the bandwidths of these two signals.

Question 2 (50 pts) – Probability of Error Analysis

Consider binary signalling with AWGN (power spectral density = $N_0/2$), in the presence of a short-circuit channel. The information bits are equally-likely; the transmission rate is $R = 1/T$ bits/sec. The transmitter filter and the matched receiver filter impulse responses are given below:

$$h_{TX}(t) = \begin{cases} A, & 0 \le t \le T \\ 0, & elsewhere \end{cases} \quad \text{and} \quad h_{RX}(t) = h_{TX}(T-t) = \begin{cases} A, & 0 \le t \le T \\ 0, & elsewhere \end{cases}.$$

The receiver filter is followed by a sampler (which samples at every T sec), which is followed by a threshold detector.

We will compare two signalling schemes in which the information bits 0 and 1 are mapped to binary a values differently to amplitude-modulate the transmitter filter.

- The first scheme, Scheme I, uses antipodal signalling. This scheme has been extensively studied in the class: The information bits 0 and 1 are mapped to -1 and 1, respectively; i.e., $\begin{matrix} 0 \rightarrow a = -1 \\ 1 \rightarrow a = 1 \end{matrix}$.

  In this scheme, the optimum threshold value of the detector is $\lambda_{th,I} = 0$. It can be shown that the average bit energy is $E_{b,av,I} = A^2T$, and the corresponding probability of error is
  $$P_{e,I} = \frac{1}{2} erfc\left(\sqrt{\frac{A^2T}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_{b,av,I}}{N_0}}\right).$$

- In Scheme II, the amplitude-modulation is performed through the following mapping:
  $\begin{matrix} 0 \rightarrow a = -0.5 \\ 1 \rightarrow a = 1.5 \end{matrix}$.

(a) By inspection, find the optimum threshold value in Scheme II: $\lambda_{th,II}$. Substantiate your decision.

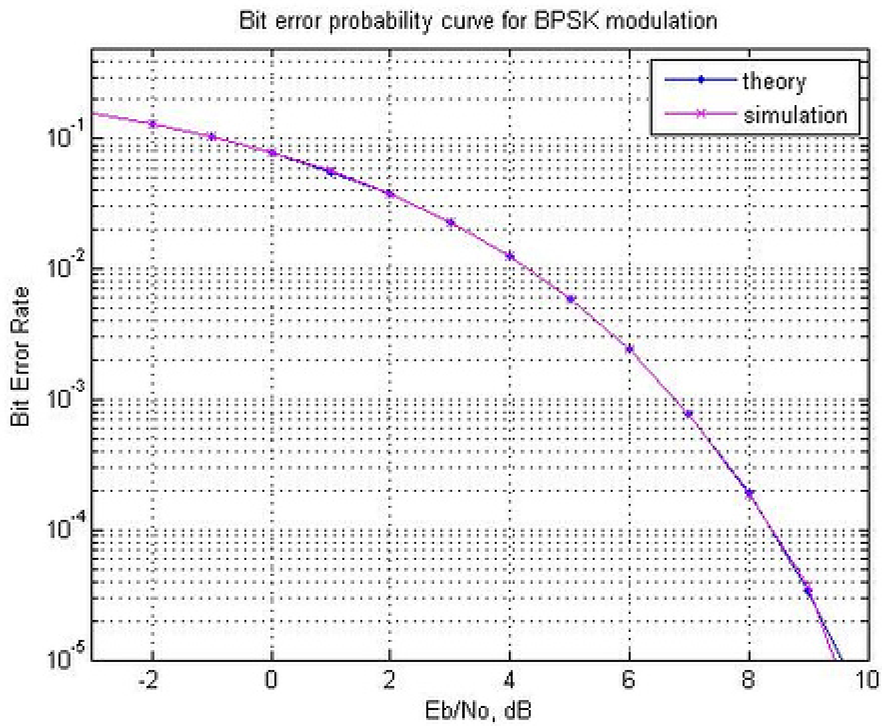
(b) Find $P_{e,II}$ in terms of A, T, and N_0. Then, express $P_{e,II}$ in terms of $E_{b,av,II}$ and N_0.

(c) Discuss which of the two schemes is better in terms of error performance by how many dBs.

Note: $erfc(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-z^2} dz$.

Question 3 (25 pts) – Rate versus Reliability

The probability of error (bit error rate) for the binary antipodal system denoted as Scheme I in Question 2 is given by $P_e = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$; this expression is plotted below.



For the proper running of an application, it is required that $P_e < 2x10^{-4}$. If the received pulse amplitude is $A = 1x10^{-6}$, find the maximum possible bit rate.

Note: The noise PSD is $N_0 = k$ x *Temp* Joules, where k is the Boltzmann constant (k= 1.38 x 10^{-23} J/K) and *Temp* is the temperature in Kelvin (assume *Temp* = 293 K, which corresponds to the room temperature).
