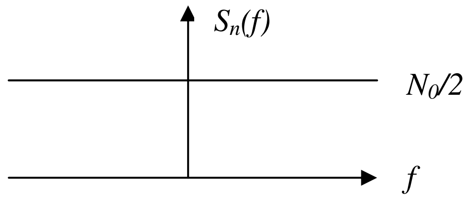
Midterm Solution

Q1)

a) Noise, inter-symbol interference (ISI) due to imperfect channel, multiuser interference.

b) The output of the system when the input is an impulse.

c)



d) $\alpha\delta(t - t_d)$

e)

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)H(f)$$

$$S_Y(f) = |H(f)|^2 S_X(f)$$

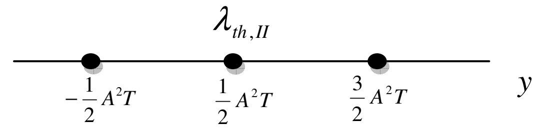
f) The bandwidth for x is 3 MHz.
The bandwidth for y is 50 MHz.

Q2)

$$s|0 = -\frac{1}{2} h_{TX}(t) * h_{RX}(t)|_{t=T} = -\frac{1}{2} A^2 T$$

$$s|1 = \frac{3}{2} h_{TX}(t) * h_{RX}(t)|_{t=T} = \frac{3}{2} A^2 T$$

a)



$$\lambda_{th,II} = \frac{1}{2}\left(\frac{3}{2}A^2T - \frac{1}{2}A^2T\right) = \frac{1}{2}A^2T$$

b)

$$P_{e,II} = P_{1|0}P_0 + P_{0|1}P_1$$

$$P_{1|0} = P(y_{|0} > \lambda_{th,II}) = P\left(-\frac{A^2T}{2} + n > \frac{A^2T}{2}\right) = P\left(n > A^2T\right)$$

n is a Gaussian random variable with zero mean and variance

$$\sigma_n^2 = \int_{-\infty}^{\infty} S_n(f)df = \int_{-\infty}^{\infty} S_w(f)\left|H_{RX}(f)\right|^2 df = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_{RX}(f)\right|^2 df = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|h_{RX}(t)\right|^2 dt = \frac{N_0}{2}A^2T$$

$$P_{1|0} = P\left(n > A^2T\right) = \int_{A^2T}^{\infty} \frac{1}{\sqrt{\pi N_0 A^2 T}} e^{-\frac{n^2}{N_0 A^2 T}} dn$$

Let $u = \frac{n}{\sqrt{N_0 A^2 T}} \Rightarrow du = \frac{dn}{\sqrt{N_0 A^2 T}} \Rightarrow$

$$P_{1|0} = P\left(n > A^2T\right) = \int_{\sqrt{\frac{A^2T}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-u^2} du = \frac{1}{2} erfc\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

Similarly,

$$P_{0|1} = P(y_{|1} < \lambda_{th,II}) = P\left(\frac{3A^2T}{2} + n < \frac{A^2T}{2}\right) = P\left(n < -A^2T\right) = P\left(n > A^2T\right) = \frac{1}{2} erfc\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

$$P_{e,II} = \frac{1}{2} erfc\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

$$E_{b,av,II} = \frac{1}{2}\left(\frac{A^2T}{4} + \frac{9A^2T}{4}\right) = \frac{5}{4}A^2T$$

$$P_{e,II} = \frac{1}{2} erfc\left(\sqrt{\frac{4E_{b,av,II}}{5N_0}}\right)$$

c) Scheme I is better than scheme II by $10\log_{10}\frac{5}{4} = 0.969\,\text{dB}$.

Q3)

$$\frac{E_b}{N_0} > 8\text{ dB} = 6.31$$

$$\frac{A^2 T}{N_0} > 6.31$$

$$R = \frac{1}{T} < \frac{A^2}{N_0 6.31} = \frac{(10^{-6})^2}{1.38\times10^{-23}\times293\times6.31} = 39.19\text{ Mbps}$$