SYSC 4600 – Fall 2010

Term Exam Solutions

Q1

$f_c = 1900$ MHz

$W = 5$ MHz

$\alpha = 0.25$

16-QAM

(a) $W = 2 \cdot \dfrac{1+\alpha}{2T_s} = \cancel{2} \cdot \dfrac{1+0.25}{\cancel{2}} R_s$

$\Rightarrow \quad 5 \text{ MHz} = 1.25 R_s$

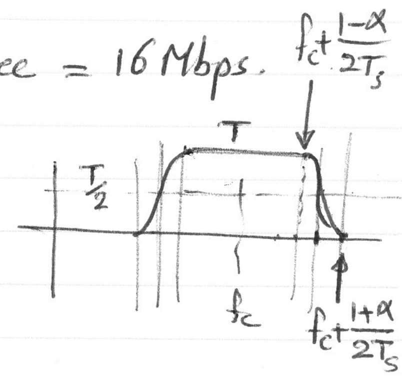
$\therefore R_s = 4$ M symb/sec. (symbol rate)

16-QAM $\Rightarrow \log_2 16$ bits/symb. $= 4$ bits/symb.

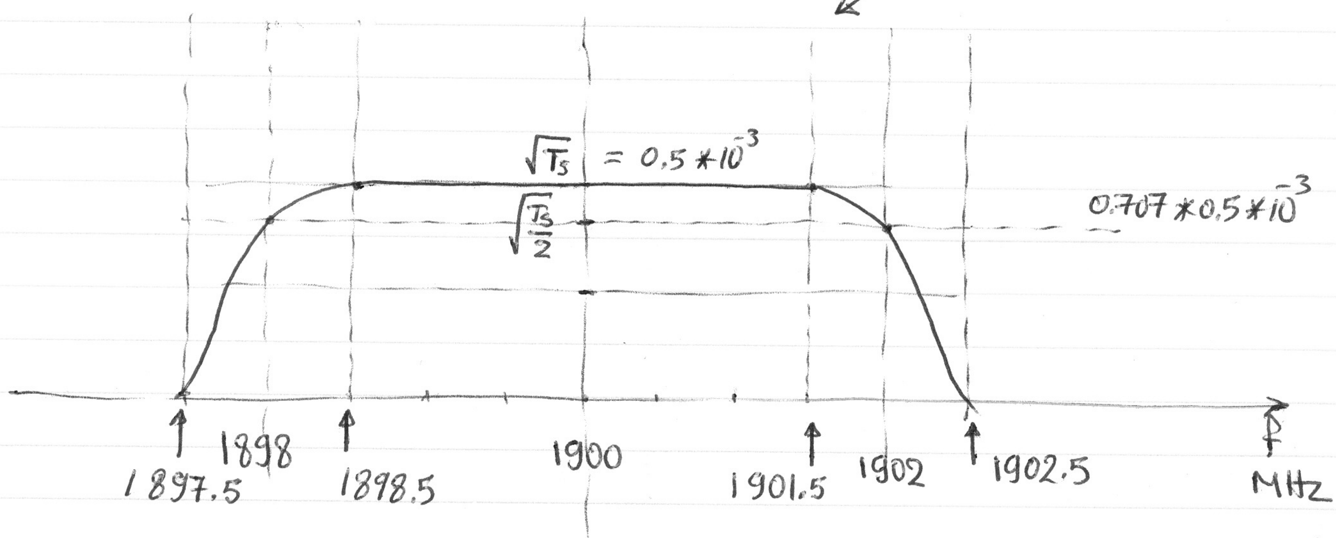
$\therefore R_b = 4$ bits/symb. $*$ 4 M symb/sec $= 16$ Mbps.

(b)

$|H_T(f)||H_R(f)| = |P_{RC}(f)|$



$\therefore |H_T(f)| = |H_R(f)| = \sqrt{|P_{RC}(f)|} = P_{SRRC}(f)$



Q2 R_b target is 1 Gbits/sec $= 10^9$ bps

$$R_{max} = n W \mu \quad \text{bits/sec}$$

for wireless $\mu_{max} \approx \log_2 64$ (64-QAM) $\rightarrow \mu_{max} = 6$ bits/symb.

OR at most $\mu_{max} = \log_7 128 \rightarrow \mu_{max} = 7$ bits/symb.

$$n = \min(n_{tx}, n_{rx})$$

at the terminal, a maximum of 4 antenna elements can be deployed $\Rightarrow$ (Note: Base stations may have more)

$$n_{max} = 4$$

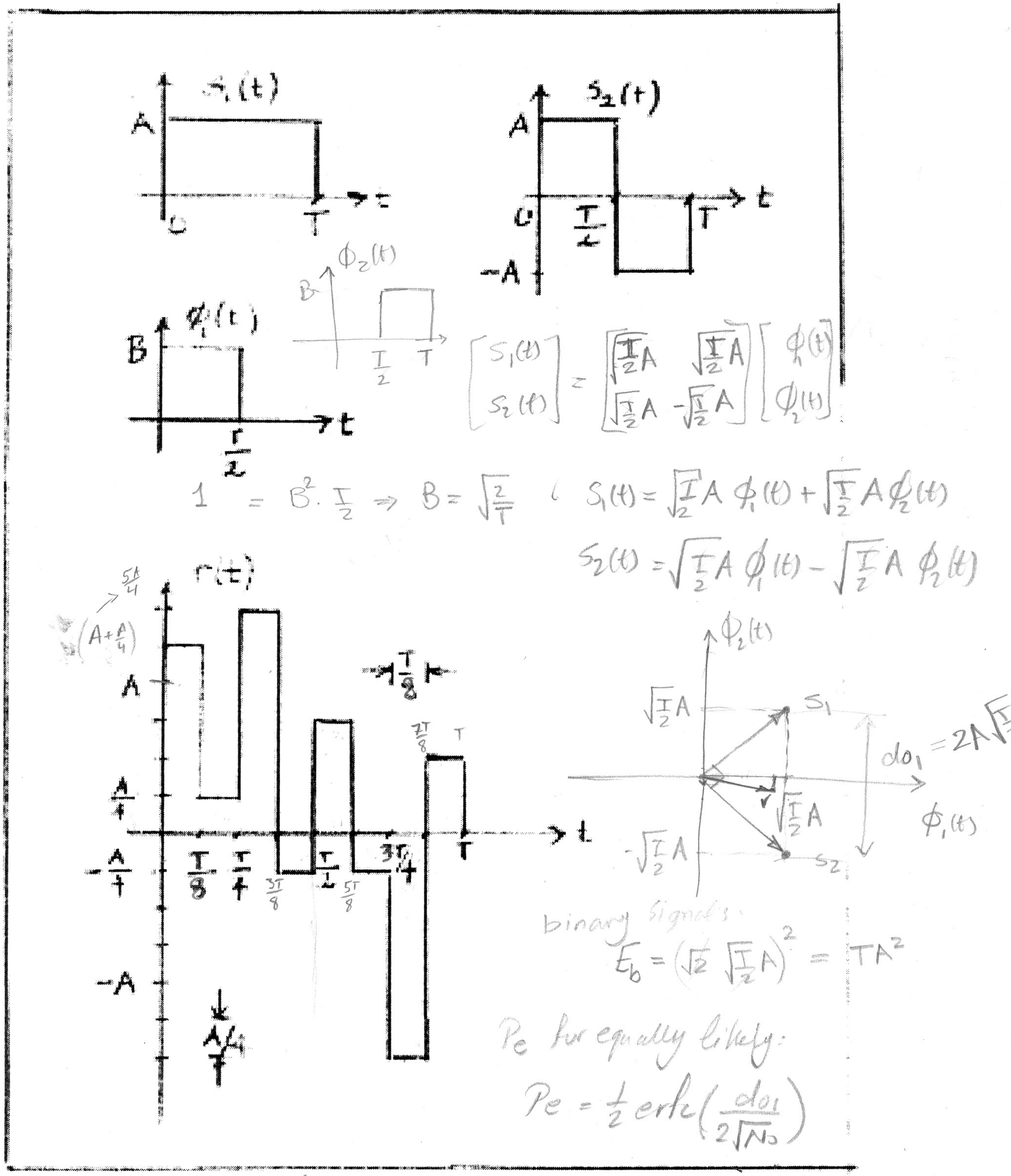
$\therefore$ we can find the required BW assuming minimum bandwidth pulse shape for the symbol (i.e., $W = R_s$ passband)

$$10^3 \text{ Mbps} = 4 * W * 7$$

$$\Rightarrow \quad W \simeq 35.71 \text{ MHz}$$

Question 3 (55 pts) – Optimum Detection

The two signals in a baseband binary signaling scheme, $s_1(t)$ and $s_2(t)$, corresponding to symbols 1 and 2, are shown in the below figure.



$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{T}{2}}A & \sqrt{\frac{T}{2}}A \\ \sqrt{\frac{T}{2}}A & -\sqrt{\frac{T}{2}}A \end{bmatrix} \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}$$

$$1 = B^2 \cdot \frac{T}{2} \Rightarrow B = \sqrt{\frac{2}{T}}$$

$$s_1(t) = \sqrt{\frac{T}{2}}A\,\phi_1(t) + \sqrt{\frac{T}{2}}A\,\phi_2(t)$$

$$s_2(t) = \sqrt{\frac{T}{2}}A\,\phi_1(t) - \sqrt{\frac{T}{2}}A\,\phi_2(t)$$

binary signals:

$$E_b = \left(\sqrt{2}\sqrt{\frac{T}{2}}A\right)^2 = TA^2$$

Pe for equally likely:

$$Pe = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{01}}{2\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{\cancel{2}A\sqrt{\frac{T}{2}}}{\cancel{2}\sqrt{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{E_b}}{\sqrt{2N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$r_1 = \langle r, \phi_1 \rangle = \int_0^T r(t)\,\phi_1(t)\,dt = \frac{T}{8}\sqrt{\frac{2}{T}}\left\{\frac{5A}{4} + \cancel{\frac{A}{4}} + \frac{3A}{2} - \cancel{\frac{A}{4}}\right\} = \frac{T}{8}\sqrt{\frac{2}{T}}\,\frac{11A}{4} = \sqrt{\frac{T}{2}}\frac{11}{16}A \qquad \sqrt{\frac{E}{2}}\,\frac{11}{16}$$

- The information bits are equally-likely.
- The source produces one bit every T seconds.
- The channel is ideal with zero-mean AWGN.
- The receiver is optimal (minimizes the probability of error).

(a) (5 pts) One of the orthonormal basis functions for the corresponding signal space, $\phi_1(t)$, is also given in the figure. Find B.

(b) (5 pts) Sketch the other basis functions.

(c) (10 pts) Write $s_1(t)$ and $s_2(t)$ in the terms of the basis functions. Draw the signal space, and show the vectors corresponding to $s_1(t)$ and $s_2(t)$.

(d) (5 pts) Write the probability of error expression in terms of bit energy, E_b, and noise power spectral density, N_0.

(e) (15 pts) Consider a received noisy signal (over one bit interval), $r(t)$, sketched also in the above figure. Draw the vector corresponding to $r(t)$ in the signal space.

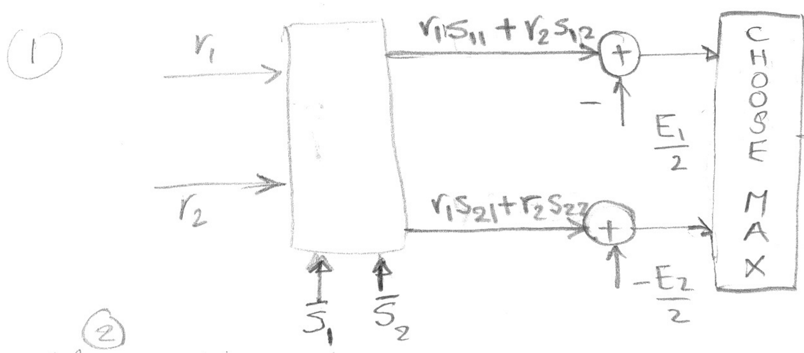
(f) (5 pts) Sketch the optimum receiver. Based on what detection principle does your receiver operate?

(g) (10 pts) Show the calculations to be performed in the receiver for detection (do those calculations). State the output of the detection (i.e., symbol 1 or 2?).

$$r_2 = \langle r, \phi_2 \rangle = \int_0^T r(t)\phi_2(t)\,dt = \frac{T}{8}\sqrt{\frac{2}{T}}\left\{\frac{3}{4}A - \frac{A}{4} - \frac{3A}{2} + \frac{A}{2}\right\} = \frac{T}{8}\sqrt{\frac{2}{T}}\left(-\frac{A}{2}\right)$$

$$= -\sqrt{\frac{T}{2}}\frac{A}{8} = -\sqrt{\frac{T}{2}}\frac{2}{16}A$$

(1)



Decision rule

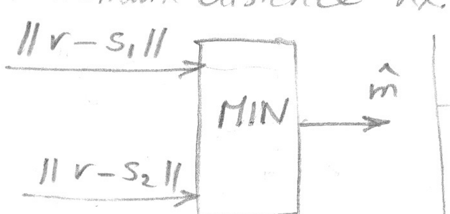
$$\langle r, s_1\rangle - \frac{E_1}{2} \underset{2}{\overset{1}{\gtrless}} \langle r, s_2\rangle - \frac{E_2}{2}$$

equal energy.

(2)

OR Minimum distance Rx. (for equally likely)

$$d(r, s_1) \underset{1}{\overset{2}{\gtrless}} d(r, s_2)$$



Calculation at Rx. (1)

$$\langle r, s_1\rangle = \int_0^T r(t)\, s_1(t)\,dt = \frac{T}{8}\frac{A}{4}A(9) = A^2 T \frac{9}{32}$$

$$\langle r, s_2\rangle = \int_0^T r(t)\, s_2(t)\,dt = \frac{T}{8}\frac{A}{4}A(13) = A^2 T \frac{13}{32}$$

$$\boxed{\hat{m} = s_2}$$