CARLETON UNIVERSITY
Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2009
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Full mark: 100 + 10 bonus points – closed-book, one-page aid-sheet allowed – 80 min.s

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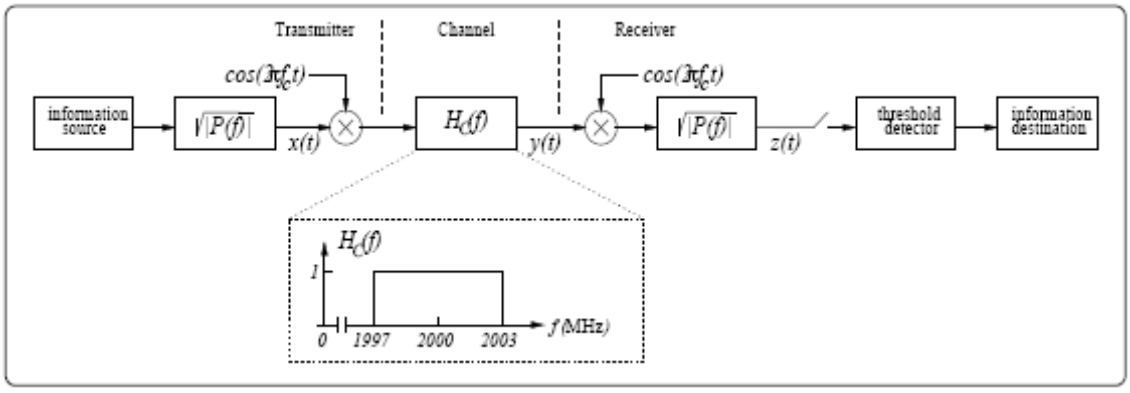
Question 1 (50 pts) – Transmission through a Bandlimited Channel

A passband communication system is given below. The transmit filter generates square-root raised-cosine pulses with α=0.5 and baseband bandwidth W MHz (including the excess bandwidth). Modulation with a sinusoidal carrier (f_c = 2000 MHz) is employed since the channel has the shape of an ideal bandpass filter.

The receiver includes a matched square-root raised-cosine (SRRC) filter with cutoff frequency W MHz. Assume that the background noise is negligible.



(a) (10 pts) Find W, by taking the bandwidth of the channel into account, that will enable ISI-free transmission at the highest possible symbol rate.

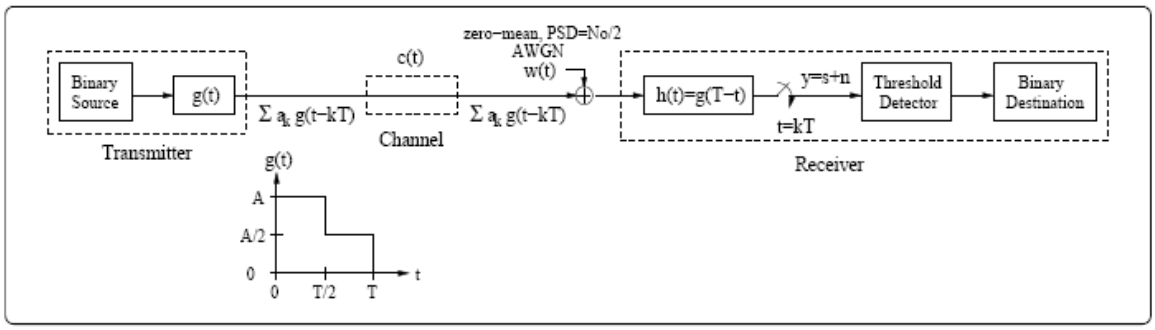
(b) (10 pts) Find the transmit bit rate if a 16-ary PAM signaling scheme is used.

(c) (10 pts) Sketch $\sqrt{|P(f)|}$.

(d) (10 pts) Let $y(t)$ denote the signal at the output of the channel. Sketch $|Y(f)|$.

(e) (10 pts) Let $z(t)$ denote the signal at the output of the receiver matched filter. Sketch $|Z(f)|$.

Question 2 (60 pts) – BER Calculation



A baseband binary signaling scheme is shown in the above figure.

- The information bits are equally-likely.
- Signaling is antipodal: $a_k = \{-1, 1\}$.
- The source produces one bit every T seconds.
- The channel is ideal with zero-mean AWGN.
- The receiver is a matched filter with $h(t)=g(T-t)$.

(a) (5 pts) Give the expression for $c(t)$, the channel impulse response. Show that

$$\left(\sum a_k\, g(t-kT)\right) * c(t) = \sum a_k\, g(t-kT),$$

where * denotes convolution.

(b) (5 pts) Find E_b, the received bit energy, in terms of A and T.

(c) (7 pts) Sketch $h(t)=g(T-t)$.

(d) (6 pts) The decision variable at the output of the sampler has a signal component s and a noise component n; that is, $y = s+n$. Find s given that a "1" is transmitted.

(e) (10 pts) Find the mean and variance of n in terms of E_b and N_o.

(f) (5 pts) Find $f_Y(y|a=-1)$ and $f_Y(y|a=1)$.

(g) (7 pts) Sketch $f_Y(y|a=-1)$ and $f_Y(y|a=1)$ together. Indicate the decision threshold.

(h) (15 pts) The probability of error is $P_e = \frac{1}{2} P(-1|a=1) + \frac{1}{2} P(1|a=-1)$. Find P_e in terms of the erfc(.) function (show the intermediate steps; you may use inspection).

[Hint: $\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-z^2} dz$]