SYSC4600: Midterm Solution
Fall 2009

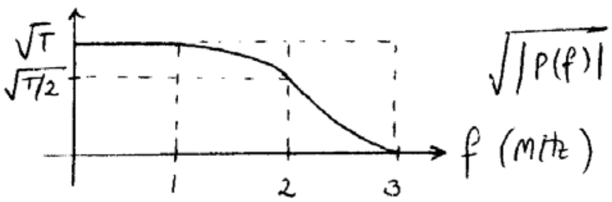
Q1 W: Baseband BW $= \dfrac{\text{Bandpass BW}}{2}$

a) Since Bandpass BW = 6 MHz, $W = 3$ MHz

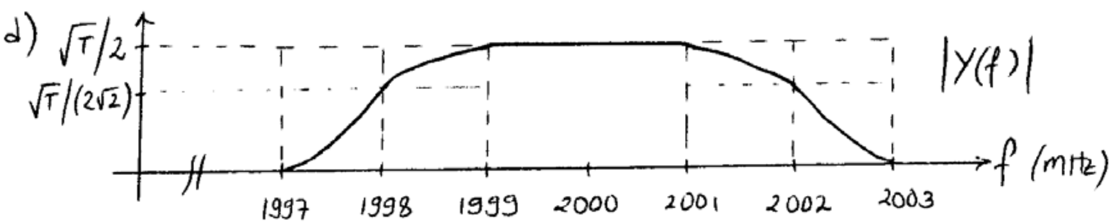
b) $R_s = \dfrac{2W}{1+\alpha} = 4$ M sym/sec

16-PAM $\rightarrow$ $\log_2 16 = 4$ symbols/bit $\rightarrow$ $R_b = 16$ Mbps

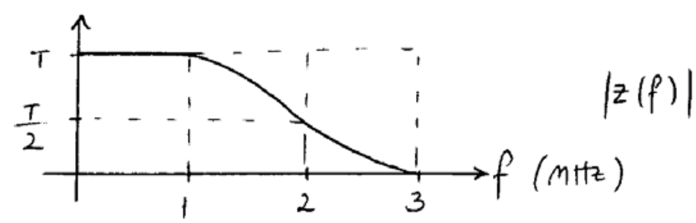
c)



d)



e)



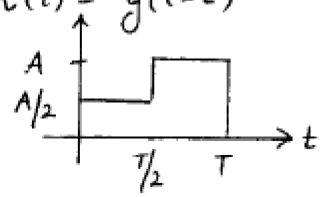
Q2 a) $c(t) = \delta(t)$

$$\left(\sum a_k g(t-kT)\right) * \delta(t) = \sum a_k \overbrace{\left(g(t-kT) * \delta(t)\right)}^{g(t-kT)} = \sum a_k g(t-kT)$$

b) $E_b \triangleq \int_0^T g^2(t)dt = A^2 \frac{T}{2} + \frac{A^2}{4}\frac{T}{2} = \frac{5}{8} A^2 T$

$h(t) = g(T-t)$

c)



d) $s = g(t) * h(t)\Big|_{t=T} = E_b$

e)

$w(t)$: zero-mean $\rightarrow n(t)$: zero-mean $\rightarrow E[n] = 0.$

$$\sigma^2 = E[n^2] - (E[n])^2$$

$$= \int_{-\infty}^{\infty} S_N(f)df = \int_{-\infty}^{\infty} S_w(f)\,|H(f)|^2\, df = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0 E_b}{2}$$

(f), (g), (h):

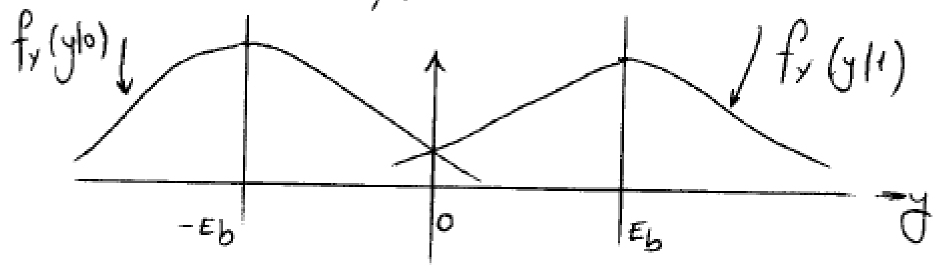
g) $y|0: \; s|0 + n = -E_b + n$

$n: G(0; \sigma_n^2) \longrightarrow y|0: G(-E_b; \sigma_n^2)$

$$f_y(y|0) = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-\frac{(y+E_b)^2}{2\sigma_n^2}}$$

$$y|1: G(E_b; \sigma_n^2) \longrightarrow f_y(y|1) = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-\frac{(y-E_b)^2}{2\sigma_n^2}}$$

h) $\lambda = (-E_b) + (E_b) / 2 = 0$



$$\frac{y + E_b}{\sqrt{2}\,\sigma_n} = z$$

$$\rightarrow P_e = \int_{\frac{E_b}{\sqrt{2}\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-z^2} \sqrt{2}\,\sigma_n \, dz = \frac{1}{2} \operatorname{erfc}\left(\frac{E_b}{\sqrt{2}\,\sigma_n}\right)$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{E_b \sqrt{2}}{\sqrt{2}\sqrt{E_b}\sqrt{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

This answer should not be surprise at all!

All antipodal signalling schemes (with matched filter receiver and AWGN) yield the same $P_e = \frac{1}{2} \operatorname{erfc}\sqrt{\frac{E_b}{N_0}}$