CARLETON UNIVERSITY

Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2008

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Full mark: 110 points – closed-book, one-page aid-sheet allowed – 80 minutes

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SOLUTIONS

(prepared by Akram Bin Sediq)

Q1)

a) The channel will not cut the transmitted signal if

$$\frac{3}{4T} = W \Rightarrow T = \frac{3}{4W}$$

$$R_s = \frac{1}{T} = \frac{4}{3}W \text{ sym/sec}$$

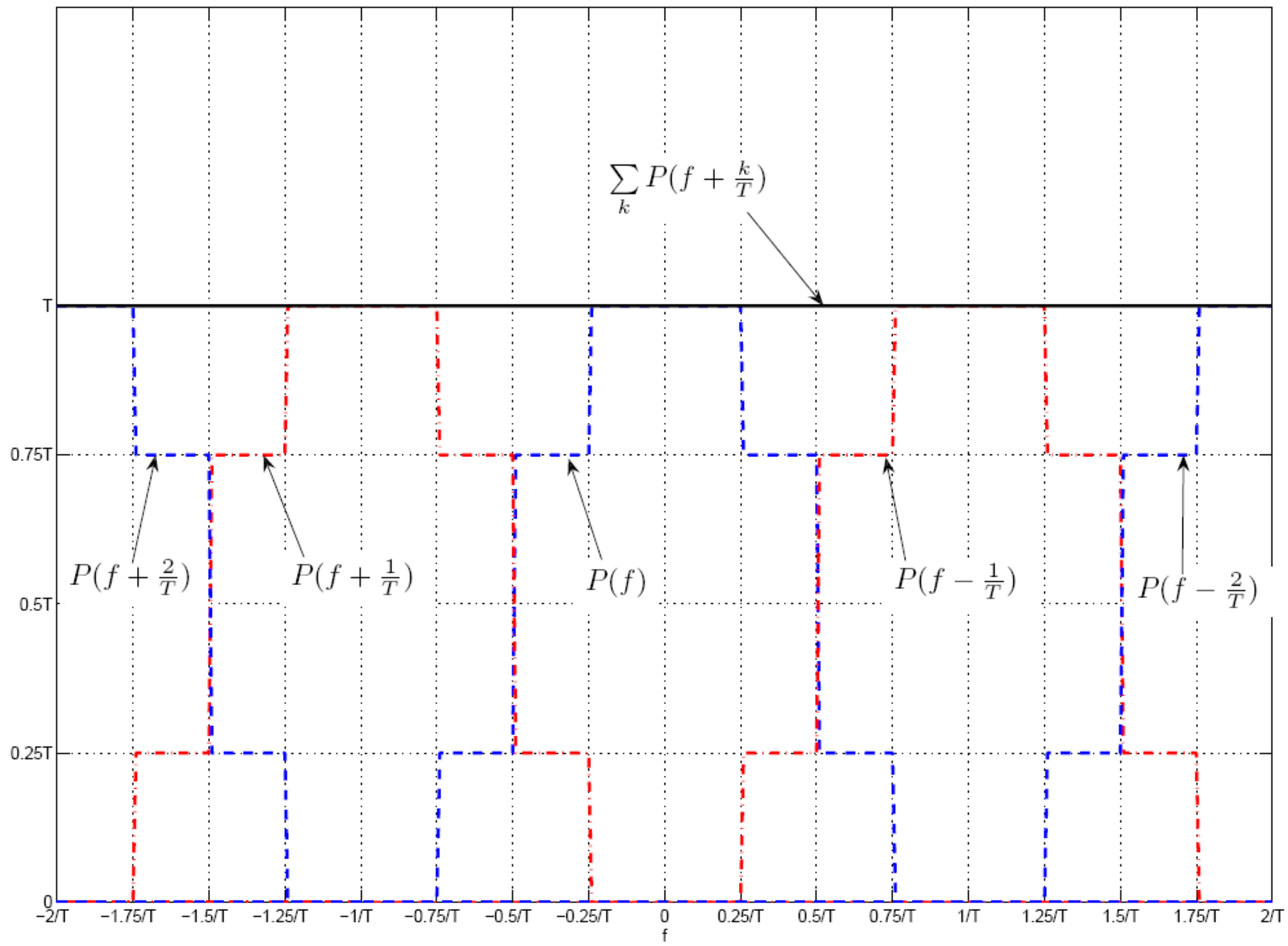
b) To have no-ISI transmission, the following condition must be satisfied:

$$\sum_k P(f + \frac{k}{T}) = \text{constant}$$

where,

$$|P(f)| = |H_T(f) H_c(f) H_R(f)|.$$

The previous condition is satisfied for the system and hence the system *does not* introduce any ISI.



The maximum symbol rate (in sym/sec) is

$$R_s = \frac{4}{3}W = \frac{4}{3}3MHz = 4\ Msym/\sec$$

The maximum bit rate is

$$R_b = (\log_2 M)\times R_s = (\log_2 16)\times R_s = 4\times 4M = 16\ Mbit/\sec$$

c)

$$R_b = (\log_2 M)\times 4M \geq 23\ Mbit/\sec \Rightarrow$$

$$\log_2 M \geq 5.75 \Rightarrow$$

$$M \geq 64$$

d) An equalizer should be added with the following transfer function

$$H_E(f) = \frac{1}{H_c(f)}$$

Q2)

a)

$$s|0 = -h_T(t) * h_R(t)\big|_{t=T} = -A^2T = -E_b$$

$$s|1 = h_T(t) * h_R(t)\big|_{t=T} = A^2T = E_b$$

$$n = w(t) * h_R(t)\big|_{t=T} = \int_{-\infty}^{\infty} w(\tau)h_R(t-\tau)d\tau = A\int_0^T w(\tau)d\tau$$

b)

$$y = s + n = \begin{cases} -E_b + n, & \text{0 was transmitted} \\ E_b + n, & \text{1 was transmitted} \end{cases}.$$

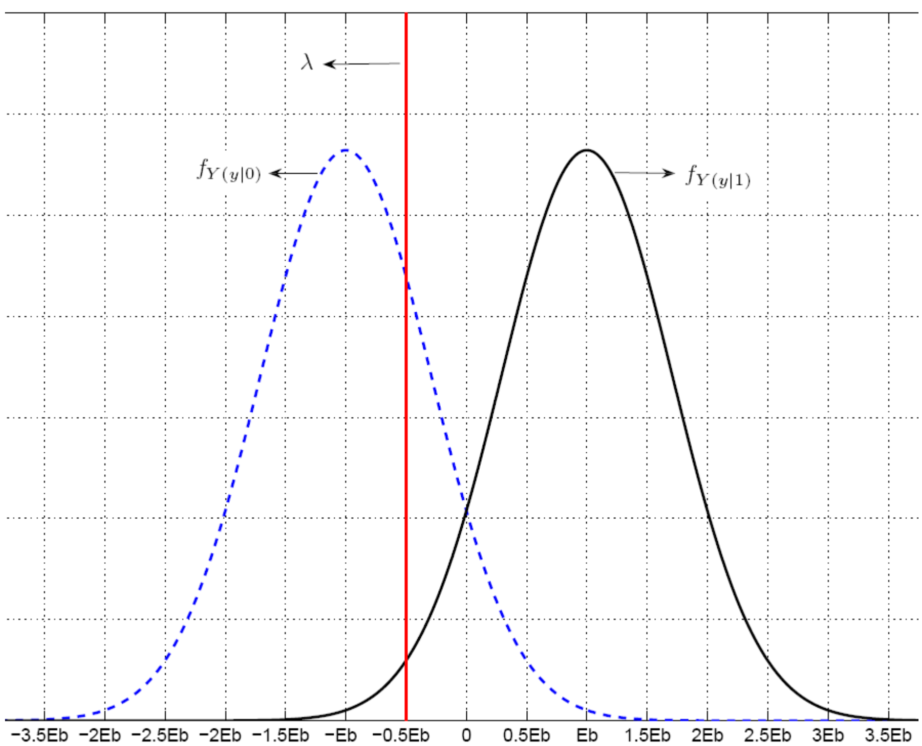
n is a Gaussian random variable with zero mean and variance σ_n^2 $(G(0,\sigma_n^2))$.

$$\sigma_n^2 = E[n^2] = R_N(0) = \int_{-\infty}^{\infty} S_n(f)df = \int_{-\infty}^{\infty} S_w(f)|H_R(f)|^2\, df$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} |H_R(f)|^2\, df = \frac{N_0}{2}\int_{-\infty}^{\infty} |h_R(t)|^2\, df = \frac{N_0}{2}E_b$$

$$y|0 \text{ is } G(-E_b, \frac{N_0}{2}E_b)$$

$$y|1 \text{ is } G(E_b, \frac{N_0}{2}E_b)$$



c)

$$P_e = P_{e|1}P_1 + P_{e|0}P_0 = 0.8P_{e|1} + 0.2P_{e|0}$$

$$P_{e|1} = \int_{-\infty}^{-E_b/2} \frac{1}{\sqrt{\pi N_0 E_b}} e^{-\frac{(y-E_b)^2}{N_0 E_b}} dy$$

Let $u = \frac{y - E_b}{\sqrt{N_0 E_b}} \Rightarrow du\sqrt{N_0 E_b} = d_y \Rightarrow$

$$P_{e|1} = \int_{-\infty}^{\frac{-3}{2}\sqrt{\frac{E_b}{N_0}}} \frac{1}{\sqrt{\pi}} e^{-u^2} dy \Rightarrow$$

$$P_{e|1} = \int_{\frac{3}{2}\sqrt{\frac{E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-u^2} dy = \frac{1}{2} erfc\left(\frac{3}{2}\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{9E_b}{4N_0}}\right)$$

Similarly,

$$P_{e|0} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

$$P_e = 0.4 erfc\left(\sqrt{\frac{9E_b}{4N_0}}\right) + 0.1 erfc\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

d)

For high $\frac{E_b}{N_0}$,

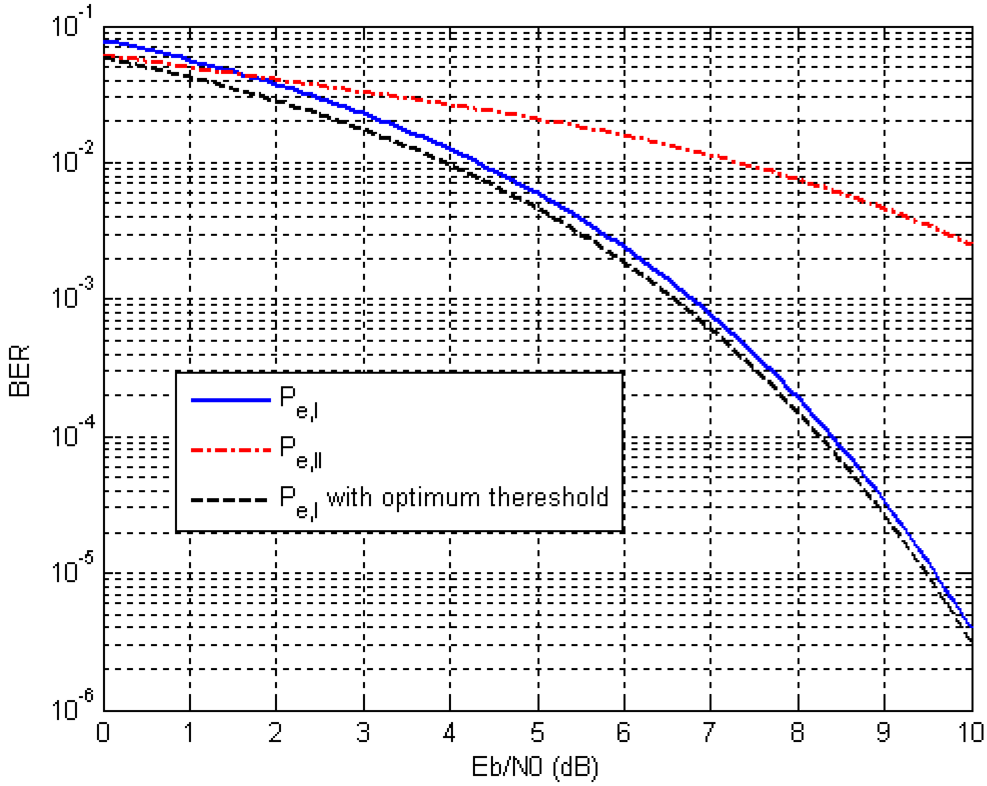
$$P_{e,I} \approx 0.1 erfc\left(\sqrt{\frac{E_b}{4N_0}}\right) > 0.5 erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \Rightarrow$$

$$P_{e,I} > P_{e,II}$$

However, if the thershold is chosen optimally to minimize $P_{e,I}$ then

$$P_{e,I} < P_{e,II}$$

This is shown in the figure below:



10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
BER
0 1 2 3 4 5 6 7 8 9 10
Eb/N0 (dB)
P_e,I
P_e,II
P_e,I with optimum thereshold