Carleton University

Dept. of Systems and Computer Engineering

Systems and Simulations—SYSC 3600 Mid Term Exam

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October 17, 2014

Instructions:

1. One double sided cheat-sheet is allowed,
2. All questions are to be answered on the examination booklet provided.
3. The total mark is 130 points—30 bonus points.
4. Unless instructed otherwise, time allowed is 2 hours.

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1. The Laplace Transform and its Properties: 20 marks total

   (a) Derive the time-differentiation property, that is, show that if $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}\{\frac{df(t)}{dt}\} = sF(s) - f(0)$. 5 marks

   (b) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$ to obtain the Laplace transform of $e^{-t}\cos(5t + \pi/4)u(t)$, where $u(t)$ is the unit step function (5 marks). What is the abscissa of convergence? (5 marks) 10 marks

   (c) Let

   $$F(s) = \frac{e^{-s^3}\cos s}{s(s+1)}.$$

   What is $f(0)$? 5 marks

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2. The Inverse Laplace Transform: 20 marks total

   (a) What is the inverse Laplace transform of the following function? 10 marks

   $$F(s) = \frac{1}{(s+1)(s+2)^2} \tag{1}$$

   (b) Use the frequency-shifting property to obtain the inverse Laplace transform of 5 marks

   $$G(s) = F(s+2), \tag{2}$$

   where $F(s)$ is given in (1).

   (c) Use the time-shifting property to obtain the inverse Laplace transform of 5 marks

   $$H(s) = e^{-5s}G(s),$$

   where $G(s)$ is given in (2).

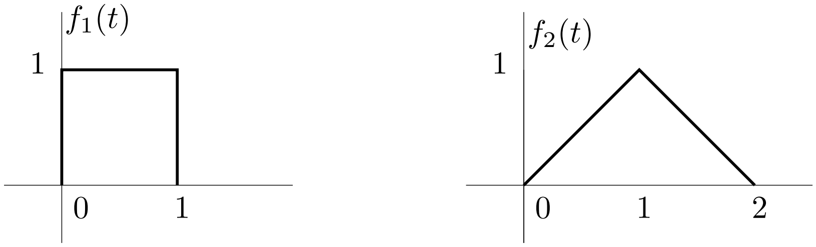

Figure 1: Functions $f_1(t)$ and $f_2(t)$ for Problem 3.

3. The Convolution Integral — 40 marks total

   Let $f_1(t)$ and $f_2(t)$ be as shown in Figure 1

   (a) Write down a mathematical expressions to describe $f_2(t)$ for different values of t. — 2 marks

   (b) Derive expressions for $f_1(t) * f_2(t)$. — 20 marks

   (c) Use the unit-step function and its shifted versions to obtain expressions for $f_1(t)$ and $f_2(t)$. — 3 marks

   (d) Obtain expressions for $F_1(s)$ and $F_2(s)$. — 5 marks

   (e) Obtain an expression for the inverse Laplace transform of $F_1(s)F_2(s)$. — 10 marks
   Hint: You will need the fact that $\mathcal{L}\{t^2u(t)\} = -\frac{2}{s^3}$.

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4. Modelling of Mechanical Systems Using Newton's Second Law of Motion: — 25 marks total

   Consider the system shown in Figure 2. In this system, the moment of inertia of the mass is $J = 1$ Nt.m.s^2, the viscosity coefficient $b = 2$ Nt.m.s, and the stiffness of the spring is $k = 2$ Nt.m. Suppose that the mass was rotated by an angle π and at time $t = 0$ it was released.

   (a) Use Newton's second law for rotational systems to obtain the differential equation that governs the motion of the system. — 15 marks

   (b) Use Laplace transform to express the differential equation as an algebraic equation in the s-domain. — 5 marks

   (c) Use the inverse Laplace transform to obtain the solution of the differential equation of the first part. — 5 marks

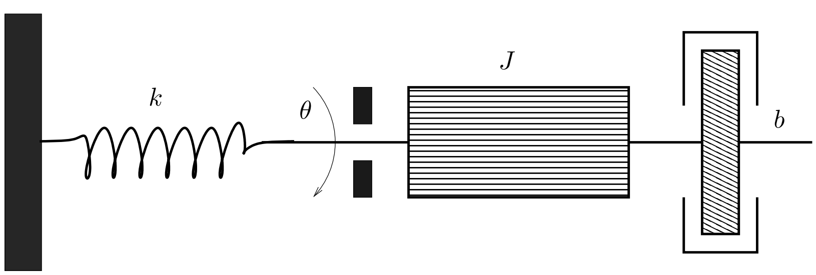

Figure 2: Rotational system with one fixed end.

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5. Modelling of Mechanical Systems Using the Law of Conservation of Energy: — 25 marks total

   Consider the system shown in Figure 3 and suppose that the radius of the pulley is 0.5 m, its mass is $M = 10$ Kg, the hanging mass is $m = 20$ Kg and the stiffness of the spring is $k = 20$ Nt.m.

   (a) Is this system conservative? Why? — 5 marks

   (b) What is the potential energy of the system? — 5 marks

(c) Knowing that the moment of inertia of the pulley is given by $J = \frac{1}{2}MR^2$, derive an expression for the kinetic energy of the system in terms of x and its derivatives. 10 marks

(d) Use the law of conservation of energy to derive an equation of motion for the system. 5 marks

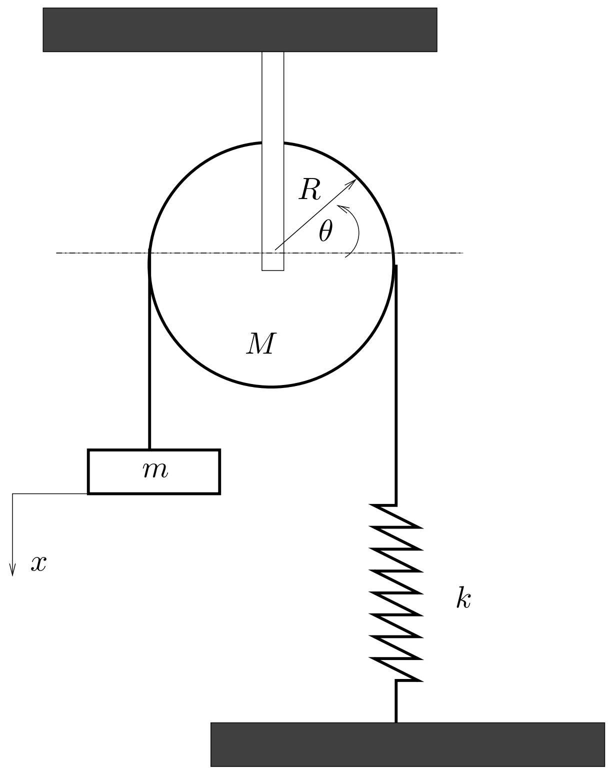

Figure 3: Mass-Pulley-Spring System.

Good luck!