# Systems and Simulations—Lecture 6

## Transient-Response Analysis of Second-Order Systems

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# Free vibration without Damping

- $m\ddot{x} + kx = 0$

# Free vibration with damping

- $m\ddot{x} + b\dot{x} + kx = 0$
- Characteristic equation—roots.
- Natural frequency $\omega_n = \sqrt{\frac{k}{m}}$
- Damping ratio $\zeta = \frac{b}{2\sqrt{km}}$.
- Mathematical model re-written:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0.$$

# Damping cases

- Underdamped: $\zeta < 1$.
- Overdamped: $\zeta > 1$.
- Critically damped: $\zeta = 1$.

# Underdamped system

- $x(t)$ decaying vibration.

# Overdamped and critically damped system

- $x(t)$ decaying exponentials.

# Unit step response

- Example: mass-damper-spring
- Example: RLC circuit.

# The End!