Lecture 5 1/8

Free vibration without damping

Free vibration with damping

Damping cases

## Systems and Simulations—Lecture 6 Transient-Response Analysis of Second-Order Systems

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Lecture 5 2/8

Free vibration without damping

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Damping cases

## Free vibration without Damping

•  $m\ddot{x} + kx = 0$ 



Lecture 5 3/8

Free vibration without damping

### Free vibration with damping

Damping cases

## Free vibration with damping

- $m\ddot{x} + b\dot{x} + kx = 0$
- Characteristic equation—roots.
- Natural frequency  $\omega_n = \sqrt{\frac{k}{m}}$
- Damping ratio  $\zeta = \frac{b}{2\sqrt{km}}$ .
- Mathematical model re-written:

$$\ddot{\mathbf{x}} + 2\zeta \omega_n \dot{\mathbf{x}} + \omega_n^2 \mathbf{x} = \mathbf{0}.$$

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## Damping cases

Free vibratior without damping

Lecture 5 4/8

Free vibration with damping

Damping cases

- Underdamped:  $\zeta < 1$ .
- Overdamped:  $\zeta > 1$ .
- Critically damped:  $\zeta = 1$ .

Lecture 5 5/8

## Free vibration without damping

Free vibration with damping

Damping cases

## Underdamped system

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• *x*(*t*) decaying vibration.

Lecture 5 6/8

Free vibration without damping

Free vibration with damping

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# Overdamped and critically damped system

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• *x*(*t*) decaying exponentials.

Lecture 5 7/8

#### Unit step response

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Free vibration without damping

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Damping cases

- Example: mass-damper-spring
- Example: RLC circuit.

Lecture 5 8/8

Free vibration without damping

Free vibration with damping

Damping cases

## The End!

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