

# Systems and Simulations—Lecture 4

## Introduction to Electrical Systems

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Fall 2014

# Elements of Electrical Systems

- Resistance elements:  $R = e_R/i$ . Heat dissipation, no energy storage.
- Capacitance elements:  $C = q/e_C$ ,  $i = C \frac{de_C}{dt}$ .
- Inductance elements:  $e_L = L \frac{di_L}{dt}$ .
- Voltage source
- Current source

Note: SI units assumed throughout.

# Fundamentals of Electrical Circuits

- Ohm's law.
- Series and parallel circuits.
- Kirchhoff's current law.
- Kirchhoff's voltage law.
- Examples.

# Modelling of Electrical Systems

- Example of RL and RC circuits.
- Example of an RLC circuit.

# Transfer Functions

- Zero initial conditions.
- Nonloading versus loading cascading elements.
- Complex impedances:
  - Resistor  $R \rightarrow R$ .
  - Capacitor with zero initial voltage:  $C \rightarrow \frac{1}{Cs}$ .
  - Inductor with zero initial current:  $L \rightarrow Ls$ .
- Transfer functions examples.

# Analogous systems

- Systems physically different by mathematically identical.
- Two mechanical-electrical analogies:
  - Force-voltage analogy: Spring-mass-damper and series RLC circuits
  - Force-current analogy: Spring-mass-damper and parallel RLC circuits

## Electromechanical systems

Modelling of a dc armature-controlled servomotor.

- Motor converts electrical to mechanical.
- With constant field current:  
 $T = K i_a$ —(Mechanical-electrical)
- Back-emf proportional to speed:  
 $e_b = K_f \dot{\theta}$ —(Mechanical-electrical)
- Motion equation:  $T = J\ddot{\theta} + b\dot{\theta}$ —(Electrical)

Gear train

- Ratio of radii = ratio of teeth:  $\frac{r_1}{r_2} = \frac{n_1}{n_2}$ .
- Equal surface speeds at point of contact:  $\omega_1 r_1 = \omega_2 r_2$ .
- Frictionless gears:  $T_1 \omega_1 = T_2 \omega_2$ .

Example.

# The operational amplifier

- Relationship between input and output: gain  $K$
- Negative feedback for stability.
- Ideal op-amp.



# Common Op-Amp Configurations

- Inverting amplifier
- Non-inverting amplifier
- Integrator circuit.