Systems and Simulations—Lecture 3

Introduction to Mechanical Systems

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Fall 2014

Some Mechanical Quantities

- Mass: Quantity of matter
- Force (Contact and Field): Cause which tends to produce change in motion of a body.
- Torque: Cause which tends to rotate a body. (Force $\times$ Arm).
- Displacement: x, Velocity: $\dot{x}$, Acceleration : $\ddot{x}$
- Work: Force $\times$ Displacement.
- Energy: Capacity to do work.
- Power: Rate of doing work.
- Inertia (for linear motion): $\frac{\text{Change in Force}}{\text{Change in Acceleration}}$
- Inertial (for rotating motion): $\frac{\text{Change in Torque}}{\text{Change in Angular Acceleration}}$.

Note: SI units assumed throughout.

Elements of Mechanical Systems

- Inertia elements. (Relates to acceleration, translating (mass) or angular (moment of inertia)
- Spring elements. (Relates to displacement, translating ($F = kx$) or angular $T = k\theta$)
- Damper elements. (Relates to velocity, translating $F = b\dot{x}$ or angular $T = b\dot{\theta}$)
- System related by:
  - Newton's laws of motion.
  - Conservation of Energy.

Mathematical Modelling of Mechanical Systems

- Rigid body assumption
- Newton's laws:
  - First law: conservation of momentum $m\dot{x}$ and $J\dot{\theta}$ constant.
  - Second law: force-acceleration or torque-angular acceleration relationships.
  - Thrid law: Action-reaction relationship.
- Conservation of energy.

Newton's Second Law

Transational motion

- Sum of forces acting through centre of mass: $\sum F = m\ddot{x}$.

Rotational motion

- Sum of torques around a fixed axis: $\sum T = J\ddot{theta}$,

where J is the moment of inertia:

$$J = \int r^2 dm.$$

Examples

- Examples on moment of inertia
- Example on rotational system
- Example on spring-mass system. Natural frequency by inspection.
- Example on spring-damper-mass.

Work, energy, power

- Work Fx (translation) and $T\theta$ (rotation)
- Energy decreases by the amount required for work done.
- Useful for systems with combinations of rotations and translations.
- Types of energy commonly encountered: potential, kinetic and dissipated.
- Power: Time rate of doing work $\frac{dW}{dt}$.

Potential Energy

- Only mass and spring can store potential energy.
- For mass at height h, $U = mgh$.
- For translational spring: $\Delta U = \int_{x_1}^{x_2} F dx = \frac{k}{2}(x_2^2 - x_1^2)$.
- For rotational spring: $\Delta U = \int_{\theta_1}^{\theta_2} T dx = \frac{k}{2}(\theta_2^2 - \theta_1^2)$

Kinetic Energy

- Only inertial elements can store kinetic energy.
- A mass travelling with velocity v has $T = \frac{1}{2}mv^2$.
- A mass rotating with angular velocity ω has $T = \frac{1}{2}J\omega^2$.
- Change in kinetic energy of a mass moving in a straight line is

$$\Delta T = \Delta W = \int_{x_1}^{x_2} F dx = \frac{1}{2}m(v_2^2 - v_1^2).$$

- Change in kinetic energy of a mass rotating about a given axis is

$$\Delta T = \Delta W = \int_{\theta_1}^{\theta_2} T d\theta = \frac{1}{2}J(\omega_2^2 - \omega_1^2).$$

Dissipated Energy

- For damper elements.
- Dissipated energy $\Delta W = \int_{x_1}^{x_2} F dx = b \int_{t_1}^{t_2} \dot{x}^2 dt$.

Conservation of Energy

- Conservative system:
  - Frictionless, no heat dissipation
  - Energy enters or leaves the system as mechanical work.

$$\Delta T + \Delta U = \Delta W$$

- Example: Mass-spring system.
- Example: Spring-cylinder system.