Systems and Simulations—Lecture 2

The Laplace Transform—A Review

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Why Laplace Transform?

- Solving linear time-invariant differential equations.
- Initial conditions already accounted for.
- Transient and steady-state responses.
- Definition and notation.

Sufficient Existence Conditions

Laplace integral converges.

- Function $f(t)$ piecewise continuous, and $\exists\sigma$ such that $e^{-\sigma t}|f(t)| \to 0$ as $t \to \infty$.
- Abscissa of convergence.
- Examples

Laplace transform examples

- $Ae^{-\alpha t}$, $Au(t)$, At, $A\sin\omega_0 t$, $A\cos\omega_0 t$.
- Translated functions. Application pulse function.
- Impulse function. Transform and relation to unit step.

Laplace transform properties

- Linearity.
- Multiplication by $e^{-\alpha t}$.
- Differentiation theorem.
- Final value theorem.
- Initial value theorem.
- Integration theorem.

Inverse Laplace transform

- Direct method: Involves contour integrations (difficult)
- Indirect methods: Use tables, or, for rational functions, use partial fraction expansion.
- Partial fraction expansion, form and factorization.
- Case with distinct poles. Examples.
- Case with multiple poles. Examples.
- Solving linear time-invariant differential equations.