Carleton University

Dept. of Systems and Computer Engineering

Systems and Simulations—SYSC 3600 Fall 2014

Homework #9

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1. Consider the system shown in Figure 1. Assume that the outputs are the displacement y and the velocity $\dot{y}$.

   - Write down the differential equation that describe the system dynamics.
   - Provide a state-space representation.
   - Use the state-space representation to obtain the transfer matrix.
   - Let $m = 1$ Kg, $b = 2$ Nt.s/m and $k = 1$ Nt/m. Obtain an explicit expression for the state-transition matrix.
   - Suppose that $y(0) = 1$ and $\dot{y}(0) = 0$ and that $p(t) = 0$. Solve the state-equation.
   - Suppose that $y(0) = 1$ and $\dot{y}(0) = 0$ and that $p(t) = u(t)$. Solve the state-equation.

2. Consider the system described by the following system of equations:

$$\ddot{w} + 2(\dot{w} - \dot{y}) + 3(w - y) + 4y = z \tag{1}$$

$$5\ddot{y} + 2(\dot{y} - \dot{w}) + 3(y - w) = 0, \tag{2}$$

   where $z(t)$ is an input force.

   (a) Let the outputs of this system be w and y. Provide a state-space representation of this system.

   (b) Use the state-space representation to obtain the state-transition matrix.

   (c) Provide a time-domain expression for the solution of the state equation assuming zero initial conditions and $z(t) = u(t)$, where $u(t)$ is the unit step function.

   (d) What is the Laplace transform of the state-transition matrix?

3. Consider the system shown in Figure 2. Assume that y is the system output. The input to the system is the displacement z.

   (a) Write down a system of differential equations to characterize the system.

   (b) Choose a set of appropriate state-variables.

   (c) Provide a state-space representation for the system.

   (d) Obtain an expression for the state-transition matrix.

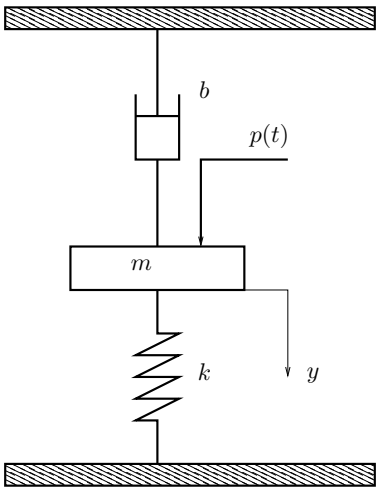

Figure 1: Mass-Damper-Spring System

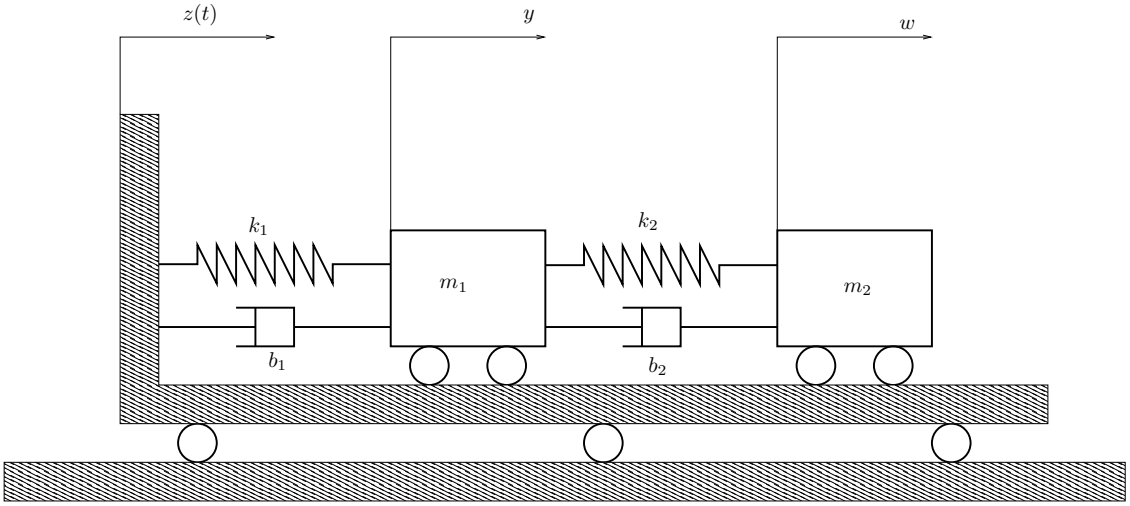

Figure 2: Cart with two masses, two springs and two dampers