Carleton University

Dept. of Systems and Computer Engineering

Systems and Simulations—SYSC 3600 Fall 2014

Homework #4

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1. Derive an expression for the moment of inertia, J, for the hollow cylinder shown in Figure 1. The expression you must obtain is a function of the inner and outer radii and the length of this cylinder.

2. Consider the system shown in Figure 2. For this system

   (a) use Newton's second law to derive a mathematical model for system dynamics;

   (b) use Laplace transform to obtain an explicit time response of the system; and

   (c) derive an expression for the natural frequency.

3. The system shown in Figure 3 is used to model the suspension of a motorcycle. The input to the system is the displacement z caused by unevenness in the road and the output is displacement x caused to the body of the driver. Let $m = 100$ Kg, $b = 200$ Nt.s/m and $k = 300$ Nt/m.

   (a) Use Newton's second law to derive a mathematical model for the system.

   (b) Obtain an expression for the transfer function.

   (c) Derive an expression for the time-response output when the input is $\cos t$.

4. Use Newton's second law to obtain a mathematical model for the second order rotational system shown in Figure 4. The input to the system is the angle θ_i and the output is the angle θ_o. Suppose that $k = 200$ Nt.m/rad, $J = 200$ Kg.m^2, and $b = 500$ Nt.m^2.s/rad. What is the time response of the system to $\theta_i(t) = u(t) - u(t - 0.1)$?

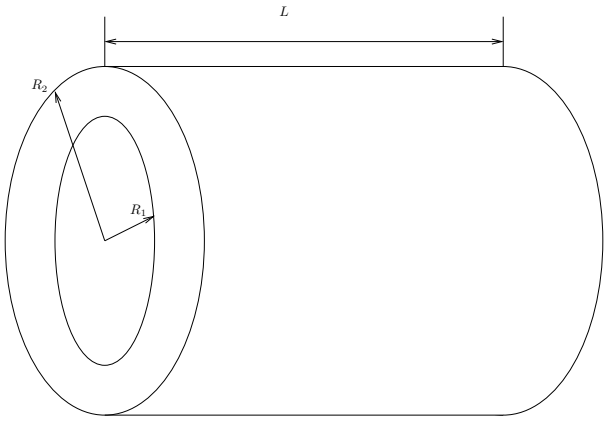

Figure 1: Hollow Cylinder

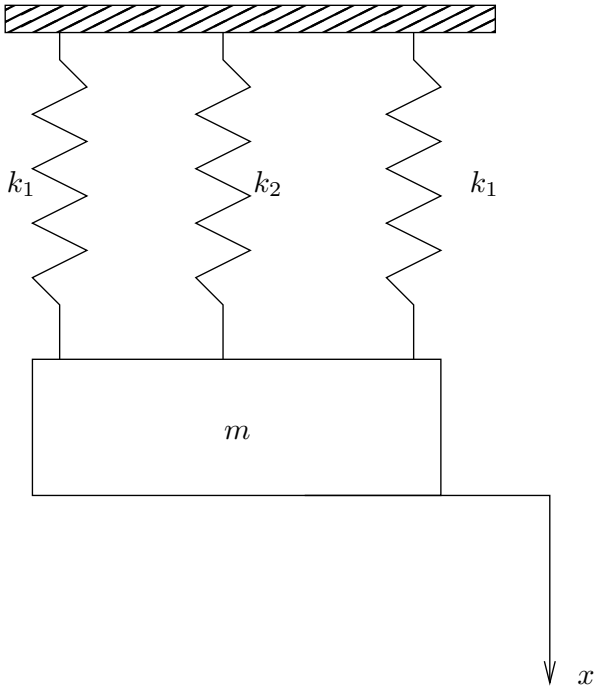

Figure 2: Hollow Cylinder

5. Consider the system shown in Figure 5. In this system, the mass $m = 40$ Kg, the pulley has a radius $R = 0.5$ m and mass $M = 20$ Kg. Suppose that $k = 200$ Nt/m and $b = 100$ Nt.s/m.

   (a) Derive an expression for the moment of inertia of the pulley.

   (b) Use Newton's second law to derive a mathematical model for the system.

   (c) Assume that $\dot{x}(0) = 0$, $x(0) = 2$ and $y(0) = 1$ and use Laplace transform to obtain an explicit expression for the displacement $x(t)$.

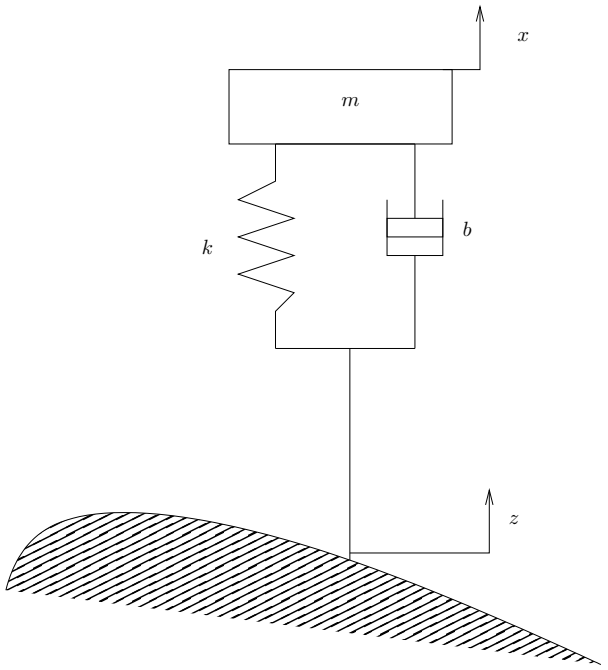

Figure 3: A model for motorcycle suspension system

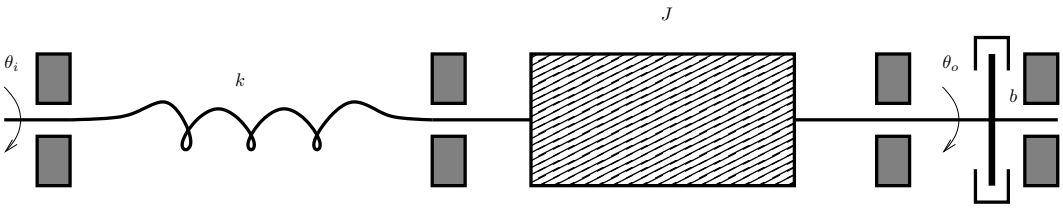

Figure 4: A second order rotational system

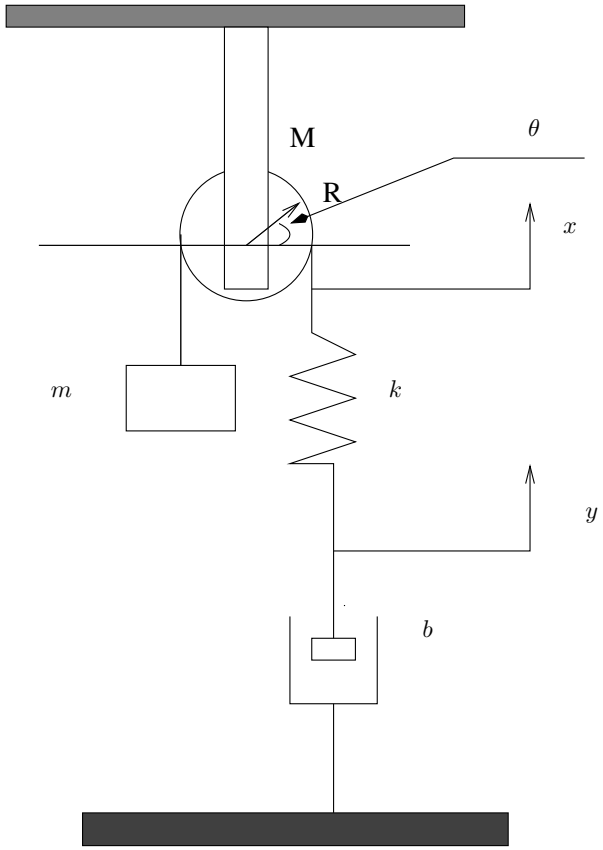

Figure 5: Second order system with pulley